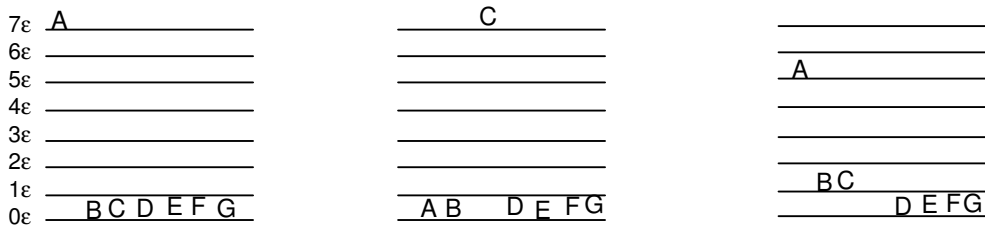


PHY2201 Summary Sheet 7

Example of Boltzmann energy sharing. 7 identical, but distinguishable systems, each with quantized energy levels  $0\varepsilon, 1\varepsilon, 2\varepsilon, 3\varepsilon \dots$ . We have a total energy of  $7\varepsilon$  to share amongst the systems. Labelling the systems A...G, some possible arrangements are :-



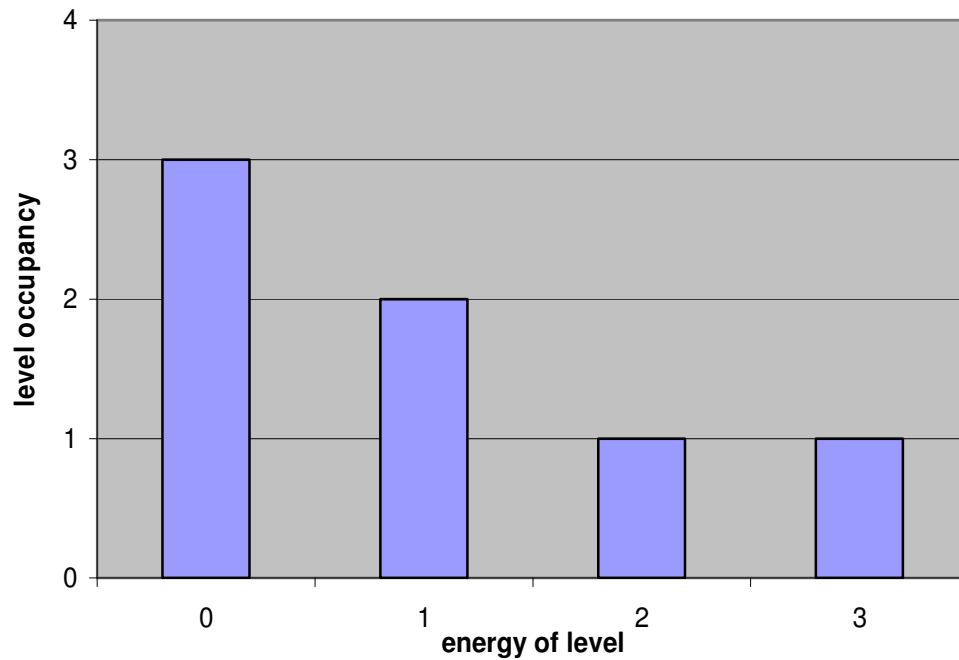
Note that the first two, distinct, arrangements nevertheless correspond to an identical macroscopic energy sharing arrangement (macrostate 'a' in the table below). Denoting the number of systems in energy level  $\varepsilon_i$  as  $n_i$ , then  $\Omega$ , the number of possible microstates corresponding to this macrostate is given by

$$\Omega = \frac{7!}{n_0!n_1!n_2!n_3!n_4!n_5!n_6! \dots} \quad \text{or in general } \Omega = \frac{N!}{\prod_{i=0}^{\infty} n_i!}$$

Since the  $n_i$ 's must satisfy the constraints  $\sum_{i=0}^{\infty} n_i = N$  and  $\sum_{i=0}^{\infty} n_i \cdot \varepsilon_i = U$ , with  $N$  the total number of systems and  $U$  the total shared energy, we can complete the table of  $\Omega$  for each macrostate

macrostate	$n_0$	$n_1$	$n_2$	$n_3$	$n_4$	$n_5$	$n_6$	$n_7$	$n_{8,9,10\dots}$	$\Omega$
a	6	0	0	0	0	0	0	1	0	7
b	5	1	0	0	0	0	1	0	0	42
c	5	0	1	0	0	1	0	0	0	42
d	4	2	0	0	0	1	0	0	0	105
e	5	0	0	1	1	0	0	0	0	42
f	4	1	1	0	1	0	0	0	0	210
g	3	3	0	0	1	0	0	0	0	140
h	2	4	0	1	0	0	0	0	0	105
i	4	0	2	1	0	0	0	0	0	105
j	3	2	1	1	0	0	0	0	0	420
k	4	1	0	2	0	0	0	0	0	105
l	1	5	1	0	0	0	0	0	0	42
m	2	3	2	0	0	0	0	0	0	210
n	3	1	3	0	0	0	0	0	0	140
o	0	7	0	0	0	0	0	0	0	1
									$\Omega_{\text{total}}$	1716

Note that, if one of the 1,716 distinct microstates were chosen at random, macrostate 'j' would occur with a probability of  $420/1716$  i.e. 24%. This is the most probable macrostate, and distributes the available energy roughly as a negative exponential function :



i.e. the relative occupancy of an energy level falls exponentially as the energy of that level increases. This pattern becomes clearer as the number of systems and the shared energy are increased.