PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2009**

1. i) Reversible process: $\Delta S_{universe} = 0$. $C_V(T_f - T_1) + C_V(T_f - T_2) = 0 \Rightarrow T_f = (T_1 + T_2)/2$.

$$\Delta S = \int_{T_1}^{(T_1+T_2)/2} \frac{C_V \, dT}{T} + \int_{T_2}^{(T_1+T_2)/2} \frac{C_V \, dT}{T} = C_V \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right] = C_V \ln\left[\frac{4T_1T_2 + (T_1-T_2)^2}{4T_1T_2}\right] > 0.$$

ii) F.T.R.: $TdS = dU + PdV = C_V dT + PdV$. For an ideal gas: $V = nRT/P$ and $C_V = C_P - nR$.
Thus, $TdS = (C_P - nR)dT + nRdT - (nRT/P)dP \quad \Rightarrow \quad \Delta S = C_P \ln(T_2/T_1) - nR\ln(P_2/P_1)$.

2. See course notes. Isotropy implies $p(-u_x) = p(u_x)$.
See course notes. Note, that the number of states with the speed from u to $u + du$ is $2\pi u \, du$.

$$\varepsilon = mu^2/2 \Rightarrow u\,du = \frac{1}{m} d\varepsilon \quad \Rightarrow \quad p(u)du = 2\frac{\alpha}{m}\exp(-2\alpha\varepsilon/m)d\varepsilon = \beta\exp(-\beta\varepsilon)d\varepsilon.$$

Boltzmann factor: $\exp(-\beta\varepsilon)$. Density of states in 2D does not depend on energy.

3. i) See course notes. In case of non-degenerate levels: $p_i = \exp\left(-\varepsilon_i/k_B T\right) \Big/ \sum_j \exp\left(-\varepsilon_j/k_B T\right)$.

$$p(\varepsilon_1)/p(\varepsilon_0) = (g_1/g_0)\exp\left(-\Delta\varepsilon/k_B T\right) = 1 \quad \Rightarrow \quad T = \frac{\Delta\varepsilon}{k_B \ln(g_1/g_0)} \approx 1.055\times10^4\,\mathrm{K}.$$

ii) $p = 6 \Big/ \left\{\frac{(6+6-1)!}{6!\,(6-1)!}\right\} = \frac{6}{462} \approx 0.013$; $S = k_B \ln 6 \approx 1.8\,k_B \approx 2.5\times10^{-23}\,\mathrm{J\,K^{-1}}$.

$p = 1/462 \approx 0.002$; $S = k_B \ln 1 = 0\,\mathrm{J\,K^{-1}}$.

4. See course notes.Use $Z = \sum_{i=0}^{\infty} \exp\left(-\frac{\varepsilon_i}{k_B T}\right)$ and the geometrical series summation rule. The factor "3" appears because the solid is three-dimensional.

$C_V = \frac{dU}{dT}$. For $T << \theta_E$, $C_V \approx 3Nk_B(\theta_E/T)^2 \exp(-\theta_E/T) \to 0$.

For $T >> \theta_E$, $C_V \approx 3Nk_B -$ classical result.

5. i) See course notes.

ii) Microscopic: $F = -k_B T \ln Z$; macroscopic: $P = -\left(\frac{\partial F}{\partial V}\right)_T$. Natural variables are *V* and *T*.

iii) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1 \Rightarrow \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V$ or $\alpha_P = -\beta_V \kappa_T$.