Differential Calculus Problem Set 2

1. A region of the xy plane is bounded by $y^2 = 3x$ and $y = x$. Make a sketch of this region and calculate its area. [5]

2. The base of a triangular sand box lies in the xy plane and is bounded by the lines $y = 1 - x$, $x = 0$, and $y = 0$. If the height of the sand in the box is given by $h(x,y) = xy$ then find the volume of sand in the box. Make a sketch of this volume. [6]

3. A semi-circular sheet in the upper half plane is bounded by $x^2 + y^2 = a^2$ and $y = 0$. The mass per unit area is given by the function $\sigma(x,y) = \sqrt{x^2 + y^2}\tan^{-1}\left(\frac{y}{x}\right)$. Find the mass of the sheet. [5]

4. Find the mass of the region corresponding to $x^2 + y^2 + z^2 \le 3$, $x \ge 0$, $y \ge 0$, $z \ge 0$, if the density is equal to xyz. [5]

5. Show that the volume of the region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the parabloid $z = x^2 + y^2$ is $\pi / 6$. [5]

6. Evaluate $\iiint\limits_{\Re} \frac{x^2\, dx\, dy\, dz}{\left(x^2 + y^2 + z^2\right)}$ where $\Re$ is the region bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$. $(b > a)$ [6]

7. Evaluate $\int\limits_{(1,1)}^{(9,3)} \left[(x + y)dx + (y - x)dy\right]$ along

   (a) the parabola $y^2 = x$,

   (b) the straight line joining the initial and final points. [6]

8. The trajectory of a particle is described by $x = at^2$, $y = bt^3$. Find the distance travelled as t increases from 0 to T [4]

9. Evaluate $\int\limits_C y^2 x\, ds$ where C is the part of the curve $x^2 + y^2 = a^2$ that lies above the x axis, begins at $\left(a/\sqrt{2}, a/\sqrt{2}\right)$ and ends at $(-a,0)$. [3]