Fourier Analysis Exercises 1.

1.

$$\int_{-L}^{L} \cos \frac{n\pi x}{L}\, dx = \frac{L}{n\pi}\left[\sin\left(\frac{n\pi x}{L}\right)\right]_{-L}^{L} = \frac{L}{n\pi}(0-0) = 0$$

$$\int_{-L}^{L} \sin \frac{n\pi x}{L}\, dx = -\frac{L}{n\pi}\left[\cos\left(\frac{n\pi x}{L}\right)\right]_{-L}^{L} = -\frac{L}{n\pi}\left((-1)^n-(-1)^n\right) = 0$$

2.

$$I_1 = \int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) dx = \int_{-L}^{L} \frac{1}{2}\left[\cos\left(\frac{(n-m)\pi x}{L}\right) - \cos\left(\frac{(n+m)\pi x}{L}\right)\right] dx$$

If $n \neq m$

$$= \frac{L}{2\pi}\left[\frac{1}{(n-m)}\sin\left(\frac{(n-m)\pi x}{L}\right) - \frac{1}{(n+m)}\sin\left(\frac{(n+m)\pi x}{L}\right)\right]_{-L}^{L}$$

$$= \frac{L}{2\pi}\left[\frac{1}{(n-m)}(0-0) - \frac{1}{(n+m)}(0-0)\right]$$

$$= 0$$

But if $n = m$

$$I_1 = \int_{-L}^{L} \frac{1}{2}\left[1 - \cos\left(\frac{(n+m)\pi x}{L}\right)\right] dx$$

$$= \frac{1}{2}\left[x - \frac{L}{\pi(n+m)}\sin\left(\frac{(n+m)\pi x}{L}\right)\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[2L - 0\right]$$

$$= L$$

$$\therefore \quad \underline{I_1 = L\,\delta_{m,n}}$$