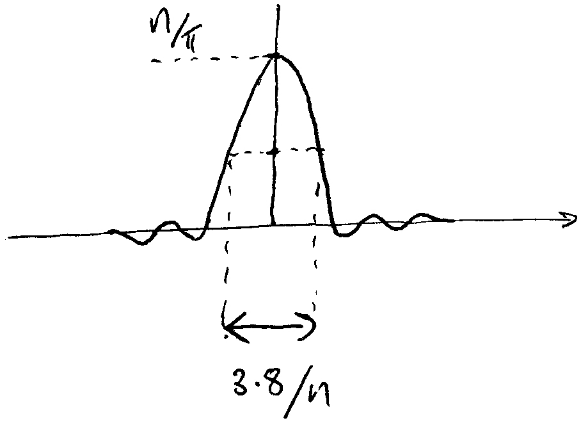
2 contd.



$$\frac{\sin nx}{\pi x}$$

$x \to 0$, $\frac{\sin nx}{\pi x} \to \frac{n}{\pi}$

For $\frac{1}{2}$ width

$$\frac{\sin nx}{\pi x} = \frac{n}{2\pi}$$

$$\sin(nx) = \frac{(nx)}{2}$$

$$nx \sim 1.9$$

3. $$I = \int_{-\infty}^{\infty} f(x)\, \delta(a(x - X))\, dx$$

By substitution, let $u = ax$

$$I = \int_{-\infty}^{\infty} f\left(\frac{u}{a}\right) \delta(u - aX) \frac{du}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} f\left(\frac{u}{a}\right) \delta(u - aX)\, du$$

$$= \frac{1}{a} f\left(\frac{aX}{a}\right) = \frac{1}{a} f(X)$$

(We implicitly assumed $a > 0$, or else limits reverse! Reversing limits back gives minus sign and hence $a \to |a|$)

But $$\int_{-\infty}^{\infty} f(x)\, \delta(x - X)\, dx = f(X)$$

$\therefore$ it seems that $\delta(a(x - X)) = \frac{1}{|a|} \delta(x - X)$