


# Exercises 5

1.   $M = \rho_0 (x^2 + y^2 + z^2)$

$$M = \int m dV = \int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2) dz dy dx$$

Do first term,

$$\begin{aligned} \text{Term 1} &= \int_0^a dy \int_0^a dz \int_0^a x^2 dx \\ &= a \cdot a \cdot \left[ \frac{x^3}{3} \right]_0^a \\ &= \frac{a^5}{3} \end{aligned}$$

$$\therefore M = 3 \times (\text{Term 1}) = \underline{a^5}$$

2. 
$$\begin{aligned} I &= \iiint_R z dV \\ &= \int_0^{2\pi} \int_0^{\pi/2} \int_0^a (r \cos \theta) r^2 \sin \theta dr d\theta d\phi \\ &= \int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{1}{2} \sin 2\theta d\theta \int_0^a r^3 dr \\ &= 2\pi \left[ -\frac{1}{4} \cos(2\theta) \right]_0^{\pi/2} \left[ \frac{r^4}{4} \right]_0^a \\ &= \cancel{2}\pi \cdot \frac{1}{\cancel{2}} \cdot \frac{a^4}{4} \\ &= \frac{\pi a^4}{4} \end{aligned}$$