Exercises 1 Solutions.

1. $x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad , z = r\cos\theta$

Consider $x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2[\sin^2\theta(\cos^2\phi+\sin^2\phi) + \cos^2\theta]$

$= r^2[\sin^2\theta + \cos^2\theta]$

$= r^2$

$\therefore \quad r = \sqrt{x^2+y^2+z^2}$

Consider $\frac{y}{x} = \frac{r\sin\theta\sin\phi}{r\sin\theta\cos\phi} = \tan\phi$

$\therefore \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

Since $z = r\cos\theta$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{z}{(x^2+y^2+z^2)^{1/2}}\right)$

2. $(\rho, \phi, z) = \left(3, \frac{\pi}{4}, 6\right)$

$(x, y, z) = (\rho\cos\phi, \rho\sin\phi, z) \quad , \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$= \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}, 6\right)$

$(r, \theta, \phi) = \left((x^2+y^2+z^2)^{1/2}, \cos^{-1}\left(\frac{z}{(x^2+y^2+z^2)^{1/2}}\right), \tan^{-1}\left(\frac{y}{x}\right)\right)$

Note ϕ has same value as in cylindrical polars!

$\phi = \frac{\pi}{4}$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2} = \sqrt{3^2+6^2} = \sqrt{45}$

$= 3\sqrt{5}$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{6}{3\sqrt{5}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) = 26.6^\circ$

$(r, \theta, \phi) = \left(3\sqrt{5}, 26.6^\circ, 45^\circ\right)$