PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY3140**
Name of module	**Methods of theoretical physics**
Date of examination	**June 2008**

1. (i) (a) Poles are at $z=\exp(i\pi/6)$, $z=\exp(i\pi/2)=i$, $z=\exp(i5\pi/6)$, $z=\exp(i7\pi/6)$, $z=\exp(i3\pi/2)=-i$ and $z=\exp(i11\pi/6)$.

(b) Upper half of z - plane: residue of $f(z)$ at the pole $z=\exp(i\pi/6)$ is $\exp(-i5\pi/6)/6$, residue at $z=\exp(i\pi/2)$ is $i/6$ and residue at $z=\exp(i5\pi/2)$ is $\exp(-i25\pi/6)/6$.

(c) $\int\limits_0^{+\infty}\frac{1}{x^6+1}\mathrm{d}x=\frac{\pi}{3}$.

(ii) Notations: $A=1$; $B=\left(1+i\sqrt{3}\right)/2=\exp(i\pi/3)$; $C=\left(-1+i\sqrt{3}\right)/2=\exp(2i\pi/3)$; $D=-1$; $F=\left(-1-i\sqrt{3}\right)/2=\exp(4i\pi/3)$; $G=\left(1-i\sqrt{3}\right)/2=\exp(5i\pi/3)$.

(a)

	A	B	C	D	F	G
A	A	B	C	D	F	G
B	B	C	D	F	G	A
C	C	D	F	G	A	B
D	D	F	G	A	B	C
F	F	G	A	B	C	D
G	G	A	B	C	D	F

(b) The unit element is $A=1$. (c) Multiplication table is symmetric ($AB=BA$, etc), therefore the group is Abelian. All elements can be expressed as B^k, where $k=0,1,2,3,4,5$, therefore, the group is cyclic. (d) The inverse element of B is G.

2. (i) $2\pi/\sqrt{a^2-1}$. Hint: Substitute $z=\exp(i\theta)$ and use contour integration.

(ii) $P=(N-M)/N$. Bookwork (definition). $x_c(4)=5/12$.

3. (a) $E=\frac{15}{16}\left(\frac{\hbar^2F^2}{m}\right)^{1/3}$. (b) $E=\frac{3}{\sqrt[3]{32}}\left(\frac{\hbar^2F^2}{m}\right)^{1/3}$. Function (a) is a better approximation.

4. (i) $\int\limits_0^{\infty}\exp\left(-x^{0.05}\right)\mathrm{d}x=20\Gamma(20)=20!$ [Hint: Substitute $x=t^{20}$].

(ii) $$E=\left(\frac{\pi}{2\sqrt{2}\gamma}\right)^{3/2}\left(\frac{\alpha\hbar^6}{m^3}\right)^{1/4}\left(n+\frac{1}{2}\right)^{3/2}\approx 1.35\times\left(\frac{\alpha\hbar^6}{m^3}\right)^{1/4}\left(n+\frac{1}{2}\right)^{3/2}.$$

Here $\gamma=\int\limits_0^1\sqrt{1-t^6}\,\mathrm{d}t\approx 0.91$.