PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2010**

1. i) See course notes.

ii) $P_{\min} = Q_C \dfrac{T_H - T_C}{T_C} = 1$ MW.

iii) Use $\mathrm{d}S_1 = \dfrac{m c_V \mathrm{d}T}{T}$ for water and $\Delta S_2 = -m c_V \dfrac{\Delta T}{T_f}$ for the reservoir, where $T_f = T_{reservoir}$.

$$\Delta S_{total} = m c_V \left[\ln\left(\frac{T_f}{T_i}\right) + \frac{T_i}{T_f} - 1 \right] \approx 131.5 \text{ J K}^{-1}$$. The sign is positive, as expected $\Delta S_{total} > 0$.

2. See course notes: $u_{mp} = 1/\sqrt{\alpha}$ $< \langle u \rangle = 2/\sqrt{\pi\alpha}$. Equilibrium: $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0 \Rightarrow$ no bulk motion..

See course notes.

$$\langle \varepsilon \rangle = \int_0^\infty \varepsilon\, p(\varepsilon)\, \mathrm{d}\varepsilon = \frac{3}{2\beta} = \frac{3k_\mathrm{B}T}{2} \Rightarrow \beta = \frac{1}{k_\mathrm{B}T}.$$

Conduction electrons – ‘quantum gas’, rules governing occupancy dominate behaviour.

3. i) See course notes; $p_i = \exp\left(-\dfrac{\varepsilon_i}{k_\mathrm{B}T}\right) \Big/ \sum_l \exp\left(-\dfrac{\varepsilon_l}{k_\mathrm{B}T}\right)$.

$$T = \frac{\Delta\varepsilon}{k_\mathrm{B} \ln 6} = 1.3 \times 10^4 \text{ K}.$$

ii) $\Omega_{n,k} = \dfrac{(n+k-1)!}{n!k!} = \dfrac{11!}{6!\,5!} = 462.$ Equally shared energy: $p = \dfrac{1}{462} \approx 0.0022$; $S = 0$.

Energy shared between two systems: $\Omega = \dfrac{6!}{3!0!3!0!0!\ldots} = 20$; $p = \dfrac{20}{462} \approx 0.043$.

4. i) See course notes: $F = U - TS$; extensive; $\mathrm{d}F = -P\,\mathrm{d}V - S\,\mathrm{d}T \Rightarrow \mathrm{d}F = -\delta W$ for $T = const$.

ii) Use $Z = \sum_{i=0}^{\infty} \exp\left(-\dfrac{\varepsilon_i}{k_\mathrm{B}T}\right)$ and the geometrical series summation rule. $F = -k_\mathrm{B}T \ln Z \approx 2.38 \times 10^{-21}$ J.

5. i) $S = k_\mathrm{B} \ln \Omega$; see course notes. $\Omega_{A+B} = \Omega_A \cdot \Omega_B \Rightarrow S_{A+B} = k_\mathrm{B} \ln \Omega_A + k_\mathrm{B} \ln \Omega_B = S_A + S_B$

a) $S = 0$; b) $S = k_\mathrm{B} \ln N = k_\mathrm{B} \ln 12 \approx 3.43 \times 10^{-23}$ JK^{-1};

c) $S = k_\mathrm{B} \ln \Omega_5 = k_\mathrm{B} \ln\left(\dfrac{N!}{5!\,(N-5)!}\right) = k_\mathrm{B} \ln\left(\dfrac{12!}{5!7!}\right) = k_\mathrm{B} \ln(792) \approx 9.21 \times 10^{-23}$ JK^{-1}.

ii) See course notes.

Classical limit: $\exp\left(\dfrac{E_i - E_\mathrm{F}}{k_\mathrm{B}T}\right) >> 1$. Therefore, $\dfrac{n_i}{w_i} \propto \exp\left(-\dfrac{E_i}{k_\mathrm{B}T}\right)$ – Boltzmann's factor.