Differential Calculus Problem Set 1

1. (a) A point P in the xy plane lies 5 units from the origin. The line OP makes an angle of 60° with the positive x axis and lies above it. Find the Cartesian coordinates (x, y) of the point P.
(b) The point Q has Cartesian coordinates $(3,7)$. Find the 2-dimensional polar coordinates of this point. [4]

2. A point P in 3 dimensional space has cartesian coordinates $(1,4,2)$. Find:
(a) the cylindrical polar coordinates of point P.
(b) the spherical polar coordinates of point P.
(c) the direction cosines of point P.

A point P in 3 dimensional space has spherical polar coordinates $(r,\theta,\phi)=\left(3,\frac{\pi}{4},\frac{\pi}{3}\right)$.
Find:
(d) The cartesian coordinates of point P
(e) The cylindrical polar coordinates of point P

[10]

3. By making Taylor expansions of u and v prove that $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v(du/dx)-u(dv/dx)}{v^2}$

[7]

4. Obtain expressions for the three first order partial derivatives of the function $f(x,y,z)=z^2\sin\left(x^2yz\right)$ [3]

5. If $z=\sin\left(x\sqrt{y}\right)$ us the Reciprocal and Reciprocity theorems to find an expression for $\left(\frac{\partial y}{\partial x}\right)_z$ in terms of x and y. (Hint: use the theorems to express $\left(\frac{\partial y}{\partial x}\right)_z$ in terms of partial derivatives of z. [5]

6. Given that $x-y=\ln(xy)$, find $\frac{dy}{dx}$ and calculate its value at the point $(1,1)$. [6]

7. Find all the second order partial derivatives of $f(x,y)=\frac{x}{y}e^{xy}$ and confirm that $f_{xy}=f_{yx}$.

[6]

8. Continuing from question 6, calculate $\frac{d^2y}{dx^2}$ and find its value at the point $(1,1)$. [8]

9. Use the Chain Rule to write the operators $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ in terms of the spherical polar coordinates (r,θ,ϕ). [14]
(begin with $\frac{\partial}{\partial x}=\left(\frac{\partial r}{\partial x}\right)_{y,z}\frac{\partial}{\partial r}+\left(\frac{\partial\phi}{\partial x}\right)_{y,z}\frac{\partial}{\partial\phi}+\left(\frac{\partial\theta}{\partial x}\right)_{y,z}\frac{\partial}{\partial\theta}$)