Exercises 8.

1. a $\int_{-10}^{10} 5x^3 \, \delta(x-4) \, dx = 5(4)^3 = \underline{320}$

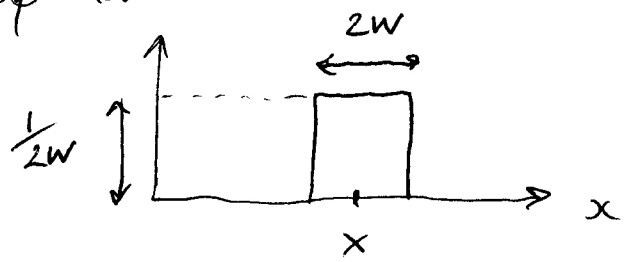
b $\int_{-10}^{0} 5x^3 \, \delta(x-4) \, dx = \underline{0}$

c $\int_{-100}^{100} 5x^3 \, \delta(x+4) \, dx = 5(-4)^3 = -320$

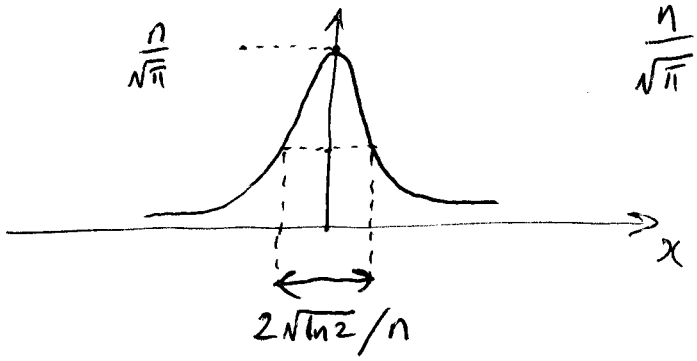
d $\int_{-100}^{100} 5x^3 \, \delta(2x-4) \, dx$

$= \int_{-100}^{100} 5x^3 \cdot \frac{1}{2} \, \delta(x-2) \, dx = \frac{5}{2}(2)^3 = \underline{20}$

2. Top hat



Gaussian:

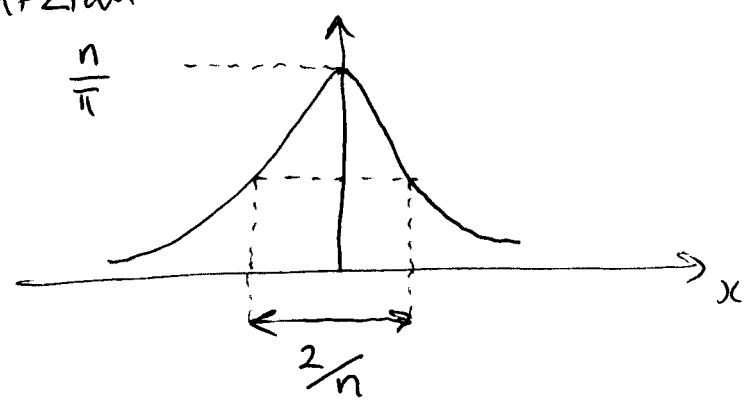


$\frac{n}{\sqrt{\pi}} \exp(-n^2x^2)$

when $\exp(-n^2x^2) = \frac{1}{2}$

$x = \frac{\sqrt{\ln 2}}{n}$

Lorentzian



$\frac{n}{\pi(1+n^2x^2)}$

For $\frac{1}{2}$ width

$\frac{1}{1+n^2x^2} = \frac{1}{2}$

$x = \frac{1}{n}$