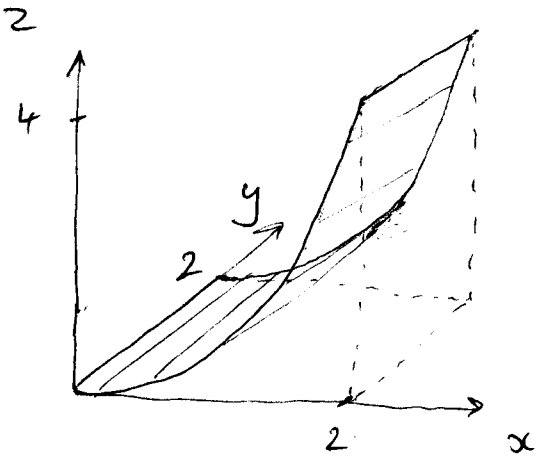
Exercises 7

1.



$$z = x^2$$

$$\frac{\partial z}{\partial x} = 2x \;,\; \frac{\partial z}{\partial y} = 0$$

$$\iint_S xy^2\, dS = \int_0^2\int_0^2 xy^2 \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$

$$= \int_0^2 y^2 dy \int_0^2 x\sqrt{1+4x^2}\, dx$$

$$= \left[\frac{y^3}{3}\right]_0^2 \left[\frac{1}{12}(1+4x^2)^{3/2}\right]_0^2$$

$$= \frac{8}{3} \times \frac{1}{12}\left[17^{3/2} - 1\right]$$

$$= \frac{2}{9}\left((17)^{3/2}-1\right) = 15.4$$

2.

$$\iint_S z\, dS = \int_0^{2\pi}\int_0^{\pi/2} (r\cos\theta)\, r^2 \sin\theta\, d\theta\, d\phi$$

where $r = a$

$$= a^3 \int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{1}{2}\sin 2\theta\, d\theta$$

$$= 2\pi a^3 \left[-\frac{1}{4}\cos 2\theta\right]_0^{\pi/2}$$

$$= \frac{2\pi a^3}{4}(1+1)$$

$$= \pi a^3$$