

4. a $f(u, v) = u^2 \ln v$ $u = x+y$, $v = x-y$

$$\left(\frac{\partial f}{\partial x}\right)_y = \left(\frac{\partial f}{\partial u}\right)_v \left(\frac{\partial u}{\partial x}\right)_y + \left(\frac{\partial f}{\partial v}\right)_u \left(\frac{\partial v}{\partial x}\right)_y$$

$$= (2u \ln v)(1) + \frac{u^2}{v}(1)$$

$$= 2u \ln v + \frac{u^2}{v}$$

$$= \frac{2(x+y) \ln(x-y) + \frac{(x+y)^2}{(x-y)}}{(x-y)}$$

$$\left(\frac{\partial f}{\partial y}\right)_x = \left(\frac{\partial f}{\partial u}\right)_v \left(\frac{\partial u}{\partial y}\right)_x + \left(\frac{\partial f}{\partial v}\right)_u \left(\frac{\partial v}{\partial y}\right)_x$$

$$= (2u \ln v)(1) + \left(\frac{u^2}{v}\right)(-1)$$

$$= 2u \ln v - \frac{u^2}{v}$$

$$= \frac{2(x+y) \ln(x-y) + \frac{(x+y)^2}{(y-x)}}{(y-x)}$$

b $f = u^2 \ln v = (x+y)^2 \ln(x-y)$

$$\left(\frac{\partial f}{\partial x}\right)_y = 2(x+y) \ln(x-y) + \frac{(x+y)^2}{(x-y)} \quad \checkmark$$

$$\left(\frac{\partial f}{\partial y}\right)_x = 2(x+y) \ln(x-y) + \frac{(x+y)^2}{(y-x)} \quad \checkmark$$