1. $y = \exp(x+y)$

Let $f := y - \exp(x+y) = 0$

then $f_x = -\exp(x+y) = -y$

$f_y = 1 - \exp(x+y) = 1 - y$

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = \underline{\frac{y}{1-y}}$$

Check: $x = \ln y - y$

$$\frac{dx}{dy} = \frac{1}{y} - 1 = \frac{1-y}{y}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \underline{\frac{y}{1-y}}$$

3. $$\frac{d^2y}{dx^2} = -\frac{\left(f_x^2 f_{yy} - 2f_x f_y f_{xy} + f_y^2 f_{xx}\right)}{f_y^3}$$

$f_{xx} = -\exp(x+y) = -y$

$f_{yy} = -\exp(x+y) = -y$

$f_{xy} = -\exp(x+y) = -y$

$$\therefore \frac{d^2y}{dx^2} = -\frac{\left[y^2(-y) - 2(-y)(1-y)(-y) + (1-y)^2(-y)\right]}{[1-y]^3}$$

$$= -\frac{\left[\cancel{-y^3} - \cancel{2y^2} + \cancel{2y^3} - y + \cancel{2y^2} - \cancel{y^3}\right]}{[1-y]^3}$$

$$= \underline{\frac{y}{(1-y)^3}}$$

Check: $\frac{dy}{dx} = \frac{y}{1-y}$ $\therefore \frac{d^2y}{dx^2} = \frac{1}{1-y}\frac{dy}{dx} + \frac{y}{(1-y)^2}\frac{dy}{dx}$

$$= \frac{1}{(1-y)^2}\frac{dy}{dx} = \underline{\frac{y}{(1-y)^3}}$$