

Exercises 2 Solutions

1. a $f = 5x^2y^3$
 $\left(\frac{\partial f}{\partial x}\right)_y = f_x = 10xy^3$, $\left(\frac{\partial f}{\partial y}\right)_x = f_y = 15x^2y^2$

b $f = e^{ax} \sin(xy)$
 $f_x = ae^{ax} \sin(xy) + ye^{ax} \cos(xy)$
 $= e^{ax} [a \sin(xy) + y \cos(xy)]$

$f_y = xe^{ax} \cos(xy)$

2. $f = 2x^2yz^3 + y^2z$
 $f_x = \left(\frac{\partial f}{\partial x}\right)_{y,z} = 4xyz^3$
 $f_y = 2x^2z^3 + 2yz$
 $f_z = 6x^2yz^2 + y^2$

3. $f = \rho \cos \varphi$, $\rho = at^2$, $\varphi = bt$

$$\begin{aligned} \frac{df}{dt} &= \left(\frac{\partial f}{\partial \rho}\right)_{\varphi} \frac{d\rho}{dt} + \left(\frac{\partial f}{\partial \varphi}\right)_{\rho} \frac{d\varphi}{dt} \\ &= \cos \varphi (2at) + (-\rho \sin \varphi) (b) \\ &= 2at \cos(bt) - abt^2 \sin(bt) \end{aligned}$$