3.

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f(x) = u(x)v(x)$$

$$\begin{cases} u(x+h) = u(x) + hu'(x) + \frac{h^2}{2!}u''(x) \\ v(x+h) = v(x) + hv'(x) + \frac{h^2}{2!}v''(x) \end{cases}$$

$$\frac{df}{dx} = \lim_{h\to 0} \frac{1}{h}\left\{ [u(x) + hu'(x) + \cdots][v(x) + hv'(x) + \cdots] - u(x)v(x) \right\}$$

$$= \lim_{h\to 0} \frac{1}{h}\left\{ \cancel{u(x)v(x)} + hvu'(x) + huv'(x) + O(h^2) - \cancel{u(x)v(x)} \right\}$$

$$= vu' + uv'.$$

4.

a

$$y = \sin[(2x+3)^2]$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{where } u = (2x+3)^2$$

$$= \cos[(2x+3)^2]\cdot[2\cdot 2\cdot(2x+3)]$$

$$= 4(2x+3)\cos[(2x+3)^2]$$

b

$$y = \sin^{-1}x$$

$$x = \sin y$$

$$\frac{dx}{dy} = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\cos y} = \frac{1}{\cos[\sin^{-1}(x)]} = \frac{1}{\sqrt{1 - \{\sin[\sin^{-1}x]\}^2}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$