Electrodynamics Revision Sheet for year 3

In Cartesian coordinates $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$

Show that if ϕ and ψ are scalar quantities and **A** and **B** are vectors then:

1. $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

2. $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

3. $\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla.\mathbf{A} + \nabla.\mathbf{B}$

4. $\nabla.(\phi\mathbf{A}) = \phi\nabla.\mathbf{A} + \mathbf{A}.(\nabla\phi)$

5. $\nabla \cdot (\mathbf{A}.\mathbf{B}) = \mathbf{A}\mathrm{x}(\nabla\mathrm{x}\mathbf{B}) + \mathbf{B}\mathrm{x}(\nabla\mathrm{x}\mathbf{A}) + (\mathbf{A}.\nabla)\mathbf{B} + (\mathbf{B}.\nabla)\mathbf{A}$

6. $\nabla \cdot (\mathbf{A}\mathrm{x}\mathbf{B}) = \mathbf{B}.(\nabla\mathrm{x}\mathbf{A}) - \mathbf{A}.(\nabla\mathrm{x}\mathbf{B})$

7. $\nabla\mathrm{x}(\mathbf{A} + \mathbf{B}) = \nabla\mathrm{x}\mathbf{A} + \nabla\mathrm{x}\mathbf{B}$

8. $\nabla\mathrm{x}(\phi\mathbf{A}) = \phi\nabla\mathrm{x}\mathbf{A} + (\nabla\phi)\mathrm{x}\mathbf{A}$

9. $\nabla\mathrm{x}(\mathbf{A}\mathrm{x}\mathbf{B}) = \mathbf{A}(\nabla.\mathbf{B}) - \mathbf{B}(\nabla.\mathbf{A}) + (\mathbf{B}.\nabla)\mathbf{A} - (\mathbf{A}.\nabla)\mathbf{B}$

10. $\nabla.(\nabla\mathrm{x}\mathbf{A}) = 0$

11. $\nabla\mathrm{x}(\nabla\phi) = 0$

12. $\nabla\mathrm{x}(\nabla\mathrm{x}\mathbf{A}) = \nabla(\nabla.\mathbf{A}) - \nabla^2\mathbf{A}$