# The ac Conductivity of a metal

Consider incident electromagnetic fields of the form:

$$\boldsymbol{E}(t) = \boldsymbol{Re}\left(\boldsymbol{E}(\omega)e^{-i\omega t}\right)$$

The equation of motion for the momentum per electron is:

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\boldsymbol{p}}{\tau} - e\boldsymbol{E} \qquad (1)$$

Seek a steady state solution of the form

$$\boldsymbol{p}(t) = \boldsymbol{Re}\left(\boldsymbol{p}(\omega)e^{-i\omega t}\right)$$

From this derive an expression for the ac conductivity of a metal.

**Solution:**

Replacing all terms in (1) we have:

$$-i\omega\boldsymbol{p}(\omega) = -\frac{\boldsymbol{p}(\omega)}{\tau} - e\boldsymbol{E}(\omega)$$

giving $$\boldsymbol{p}(\omega) = -\frac{e\boldsymbol{E}(\omega)}{1/\tau - i\omega}$$

Thus the current density $\boldsymbol{j}$ is given by:

$$\boldsymbol{j} = -\frac{ne\boldsymbol{p}}{m} = \frac{ne^2\boldsymbol{E}(\omega)}{m\left(1/\tau - i\omega\right)} = \sigma(\omega)\boldsymbol{E}$$

Therefore

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \qquad \text{with } \sigma_0 = \frac{ne^2\tau}{m}$$