Electromagnetic Waves Problem

Consider a circularly polarized wave of the form:

$$\boldsymbol{E}_1 = \boldsymbol{E}_{x0}\sin(kz-\omega t)+\boldsymbol{E}_{y0}\cos(kz-\omega t),$$

$$\text{with } |\boldsymbol{E}_{x0}| = |\boldsymbol{E}_{y0}|$$

Suppose this is then added to its counterpropagating equivalent:

$$\boldsymbol{E}_2 = \boldsymbol{E}'_{x0}\sin(kz+\omega t)+\boldsymbol{E}'_{y0}\cos(kz+\omega t)$$

$$\text{with } |\boldsymbol{E}'_{x0}| = |\boldsymbol{E}'_{y0}|$$

What is the resultant waveform if the wave is being reflected from a rigid wall? Describe it.

Solution:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2$$

$$\begin{aligned}
&= \boldsymbol{E}_{x0}\sin(kz)\cos(\omega t)-\boldsymbol{E}_{x0}\cos(kz)\sin(\omega t)\\
&+\boldsymbol{E}_{y0}\cos(kz)\cos(\omega t)+\boldsymbol{E}_{y0}\sin(kz)\sin(\omega t)\\
&+\boldsymbol{E}'_{x0}\sin(kz)\cos(\omega t)+\boldsymbol{E}'_{x0}\cos(kz)\sin(\omega t)\\
&+\boldsymbol{E}'_{y0}\cos(kz)\cos(\omega t)-\boldsymbol{E}'_{y0}\sin(kz)\sin(\omega t)
\end{aligned}$$

For the wave to have no amplitude at the wall (which we label z =0) at all times requires that $\boldsymbol{E}_{x0} = \boldsymbol{E}'_{x0}$ and $\boldsymbol{E}_{y0} = -\boldsymbol{E}'_{y0}$. Therefore

$$\boldsymbol{E} = 2\boldsymbol{E}_{x0}\sin(kz)\cos(\omega t)+2\boldsymbol{E}_{y0}\sin(kz)\sin(\omega t)$$

$$= 2\left[\boldsymbol{E}_{x0}\cos(\omega t)+\boldsymbol{E}_{y0}\sin(\omega t)\right]\sin(kz)$$

This is simply a spiral in space that rotates in time with an angular frequency ω.