Electromagnetic Waves Problem

Consider a circularly polarized wave of the form:

$$\boldsymbol{E}_1 = \boldsymbol{E}_{x0}\sin(kz-\omega t)+\boldsymbol{E}_{y0}\cos(kz-\omega t),$$

$$\text{with } |\boldsymbol{E}_{x0}| = |\boldsymbol{E}_{y0}|$$

Suppose this is then added to its counterpropagating equivalent:

$$\boldsymbol{E}_2 = \boldsymbol{E}_{x0}\sin(kz+\omega t)+\boldsymbol{E}_{y0}\cos(kz+\omega t)$$

What is the resultant waveform?

Describe it.

Solution:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2$$

$$\begin{aligned}
&= \boldsymbol{E}_{x0}\sin(kz)\cos(\omega t)-\boldsymbol{E}_{x0}\cos(kz)\sin(\omega t)\\
&+\boldsymbol{E}_{y0}\cos(kz)\cos(\omega t)+\boldsymbol{E}_{y0}\sin(kz)\sin(\omega t)\\
&+\boldsymbol{E}_{x0}\sin(kz)\cos(\omega t)+\boldsymbol{E}_{x0}\cos(kz)\sin(\omega t)\\
&+\boldsymbol{E}_{y0}\cos(kz)\cos(\omega t)-\boldsymbol{E}_{y0}\sin(kz)\sin(\omega t)
\end{aligned}$$

$$= 2\boldsymbol{E}_{x0}\sin(kz)\cos(\omega t)+2\boldsymbol{E}_{y0}\cos(kz)\cos(\omega t)$$

$$= 2\left[\boldsymbol{E}_{x0}\sin(kz)+\boldsymbol{E}_{y0}\cos(kz)\right]\cos(\omega t)$$

This is simply a spiral in space that oscillates in time with an angular frequency ω.