Problem on Electromagnetic Energy Density

A long coaxial cable carries current *I* (the current flows along an inner cylinder of radius *a* and back along an outer cylinder of radius *b*).

What is the magnetic energy stored in a section of length *l*?

Solution:

According to Ampere's law, the field between the cylinders is given by:

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

Elsewhere the field is zero.

Thus the energy per unit volume is given by;

$$\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi s}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 s^2}$$

Then the energy in a cylindrical shell of thickness *ds* is given as:

$$\left(\frac{\mu_0 I^2}{8\pi^2 s^2}\right)2\pi l s ds = \frac{\mu_0 I^2 l}{4\pi}\left(\frac{ds}{s}\right)$$

Integrating from *a* to *b* then gives:

$$W = \frac{\mu_0 I^2 l}{4\pi} ln\left(\frac{b}{a}\right)$$