Problem on Electromagnetic Waves

The electric field of a standing electromagnetic plane wave in empty space is given by:

$$\boldsymbol{E}_y(x,t) = 2\boldsymbol{E}_{y0} \sin kx \cos \omega t$$

Derive an expression for the magnetic induction ***B*** (*x*, *t*)

Solution:

Maxwell

$$\Delta \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Since ***E*** is only in the *y* direction and the only spatial variation of ***E*** is with *x* (through sin *kx*), then this equation reduces to:

$$\frac{\partial \boldsymbol{E}}{\partial x}\hat{z} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Integrate to obtain ***B***.

$$\boldsymbol{B}_z(x,t) = -\int \frac{\partial \boldsymbol{E}}{\partial x} dt = -2\boldsymbol{E}_{y0} k \cos kx \int \cos \omega t dt = -\frac{2\boldsymbol{E}_{y0} k}{\omega} \cos kx \sin \omega t$$

Therefore:

$$\boldsymbol{B}_z(x,t) = -2\boldsymbol{B}_{z0} \cos kx \sin \omega t$$

With

$$\boldsymbol{B}_{z0} = \frac{\boldsymbol{E}_{y0} k}{\omega} = \frac{\boldsymbol{E}_{y0}}{c}$$