Problem on EMF

A uniform magnetic induction **B** (t) is pointing straight up the z axis, filling the shaded region in the figure. If the magnitude of **B** is changing at the rate dB/dt find an expression for the induced E field at radius s.

Solution:

By symmetry **E** can only point in one of three directions. It could be along z, clearly impossible because **E** and d**B**/dt are related by a curl equation.

It may point radially or tangentially (circumferentially) in the plane orthogonal to **B**. If it points radially then how does it end? There are no charges.

Thus **E** has to be tangential.

Now draw a loop of radius s in the plane orthogonal to **B**.

Apply Faraday's law.

$$\oint \boldsymbol{E} \cdot \boldsymbol{dl} = \mathrm{E}(2\pi \mathrm{s}) = -\frac{d\varphi}{dt} = -\frac{d}{dt}\left(\pi s^2 B(t)\right) = -\pi s^2 \frac{dB}{dt}$$

Therefore

$$E = -\frac{s}{2}\frac{dB}{dt}\hat{\boldsymbol{\varphi}}$$

If **B** is increasing then **E** is clockwise from above.