Problem on magnetostatics

Find the magnetic induction B at a distance z above the centre of a circular loop of radius R, which carries steady current of magnitude I.

Solution:

Each small segment of the wire has current ***dI***, where ***I*** points in the direction of the tangent to the wire and ***dI*** has magnitude given by the length of the small segment *dl* multiplied by *I*.

The small field of such a segment is given by ***d*B**. This is at right angles to ***dI***. As we sweep around the loop so ***d*B** sweeps out a cone. The horizontal components of B cancel, leaving only the vertical component.

$$B(\mathrm{z}) = \frac{\mu_0}{4\pi} I \int \frac{dl}{r^2} \cos\theta$$

Now r is a constant magnitude as is cos θ. Therefore the integral is essentially the integral of dl, which is 2πR.

So $$B(\mathrm{z}) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right) = \frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+z^2\right)^{3/2}}$$