

Magnetism (Part II)

In this lecture

- ★ Magnetic field
- ★ Magnetic force on a moving charge
- ★ Magnetic Field Lines
- ★ Magnetism for MRI

Magnetic Field

Just as there is a field associated with electric charge there is a field associated with a magnet

Electric Field, E

- Collection of electric charge at rest creates an *electric field*, E
- Field exerts force $F=q \times E$ on other charge, q

Magnetic Field

Just as there is a field associated with electric charge there is a field associated with a magnet

Magnetic Field, B

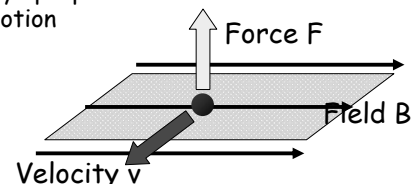
- A moving charge (or current) creates a *magnetic field*, B
- Field exerts force on any other moving charge in the field

Moving Charges in a B-Field

- Characteristics of force on a charge moving in a magnetic field
- Magnitude of force proportional to
 1. Size of charge
 2. 'Strength' of magnetic field
 3. Speed of moving charge!

Moving Charges in a B-Field

- Characteristics of force on a charge moving in a magnetic field
- Direction of force
 - Always perpendicular to both B and direction of motion



Moving Charges in a B-Field

- Magnitude of force is proportional to the component of velocity perpendicular to the B field

$$\mathbf{F} = q \times \mathbf{v}_{\perp} \times \mathbf{B}$$

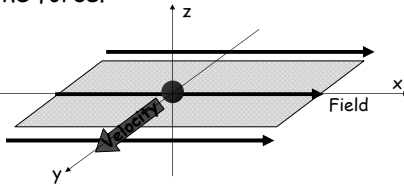
- When that component is zero (i.e. \mathbf{v} and \mathbf{B} are parallel or anti-parallel) the force is zero

Magnetic Field Strength

- SI units of magnetic field strength is the Tesla (T)
- One Tesla = 1 N/A m
- Older unit: gauss (G)
- One Tesla = 10,000 gauss
- Earth's magnetic field strength:
 - Approximately 50 μ T at equator & 100 μ T at poles
 - (Magnetic door latch ~100 mT)

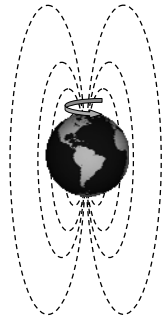
Example

- A beam of protons moves through a uniform magnetic field of 2T, directed along the x-axis. The protons are travelling along the y-axis at with a velocity of $3 \times 10^5 \text{ ms}^{-1}$. What is the magnitude of the force?
- What would change if an electron beam was used instead of a proton beam?



Magnetic Field Lines

- As with electric field magnetic field can be represented by field lines
- Unlike electric field lines, they do NOT point in the direction of the force on a charge
- They point along the direction that a compass needle would point
- Density of lines proportional to field strength
- Field lines never intersect!

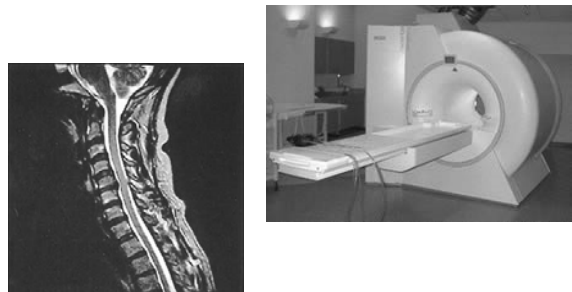


Magnetic Field Lines

- Common sources of magnetic fields
 - Permanent bar magnet
 - Solenoid
 - Iron core electromagnet
 - Straight wire
 - Loop of wire

MRI (Magnetic Resonance Imaging)

Nuclear Magnetic Resonance (NMR)



Protons in a Magnetic Field

- Not all nuclei exhibit NMR
- Must have odd number of nucleons and therefore exhibit magnetic component or a **MAGNETIC MOMENT**

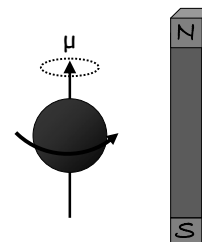
^1H - Hydrogen

^{13}C - Carbon

^{31}P - Phosphorous

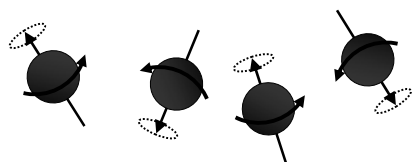
Protons in a Magnetic Field

- Hydrogen proton can be compared to a bar magnet
- Magnetic moment, μ



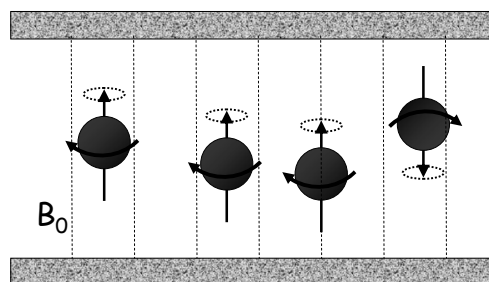
Protons in a Magnetic Field

- In the absence of B-field protons can adopt ANY orientation



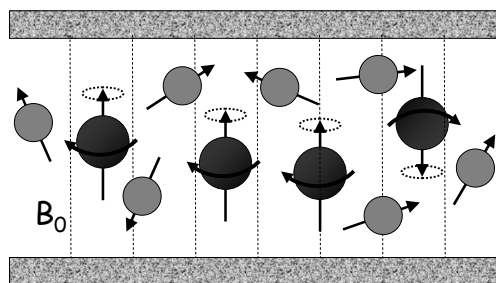
Protons in a Magnetic Field

- Switch on B-field



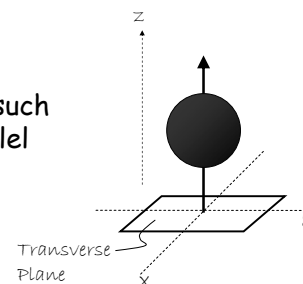
Protons in a Magnetic Field

- Switch on B-field



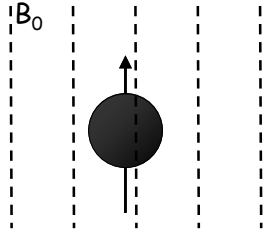
3-Dimensional Co-ordinate System

- Axes x, y, z
- Define axes such the z is parallel with B_0



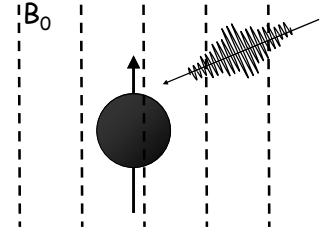
Precession

- Spins align with B-field



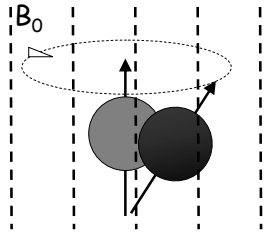
Precession

- Additional energy from RF pulse



Precession

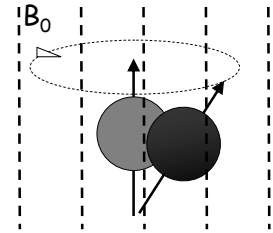
- Spins rotate about mean position
- Analogous to spinning top in gravitational field



Precession

$$\omega_L = \gamma B_0$$

- Frequency of rotation known as **Larmor frequency**, ω_L
- Governed by strength of magnetic field and property of spin called **gyromagnetic ratio**, γ



Precession

Gyromagnetic Ratio

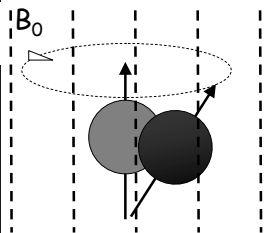
$$\gamma = \frac{\text{Magnetic Moment}}{\text{Spin Angular Momentum}}$$

$$\gamma = \frac{\mu}{2\pi L}$$

$$\gamma = \frac{1.41 \times 10^{-26} \text{ JT}^{-1}}{2\pi \times 0.527 \times 10^{-34} \text{ Js}^{-1}}$$

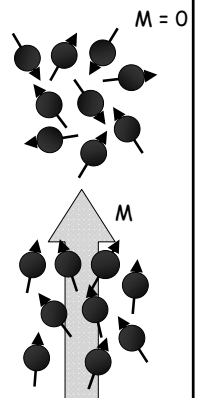
$$\gamma = 42.58 \text{ MHz T}^{-1}$$

$$\omega = \gamma B_0$$



Magnetisation Vector

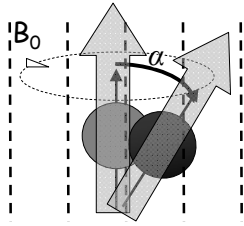
- Magnetisation Vector, M
- Represents the net magnetic moment or the 'sum behaviour' of all protons in sample



Magnetisation Vector

Flip Angle

- Depends upon pulse energy
- B_1 = B-field associated with pulse
- t_p = pulse duration
- Angle controlled by varying pulse time or amplitude



$$\alpha = \gamma B_1 t_p$$

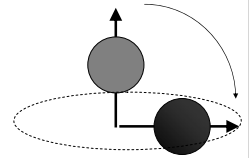
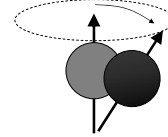
- Magnetisation Vector is flipped
- Not protons

Magnetisation Vector

Proton in B_0
Maximum M_z
Zero $M_{x,y}$

RF Pulse
 M_z
 M_{xy}

Larger RF Pulse
Zero M_z
Maximum $M_{x,y}$



Summary

- ★ Magnetic field
- ★ Magnetic force on a moving charge
- ★ Magnetic Field Lines
- ★ Magnetism for MRI

Practice Questions

PAM2011: Lecture 8 Problem Sheet

1. Magnetic fields in excess of 5 gauss can interfere with cardiac pacemakers. How many mT is this?
2. A beam of protons moves through a uniform magnetic field of 4T, directed along the positive x-axis. The protons are travelling along the y-axis at with a velocity of $1.5 \times 10^5 \text{ ms}^{-1}$. What is the magnitude of the force and along which direction does the force act?
3. A beam of electrons moves through a uniform magnetic field of 1T, directed along the positive x-axis. The protons are travelling at an angle of 30° to the y-axis at with a velocity of $1 \times 10^5 \text{ ms}^{-1}$. What is the magnitude of the force and along which direction does the force act?
4. An electron beam travelling at a velocity of $1 \times 10^6 \text{ ms}^{-1}$ through a magnetic field of 1T experiences a force of $4 \times 10^{-14} \text{ N}$. What is the angle between the direction of the electron beam and the magnetic field?