Solutions of Problems 2 “Stars from birth to death” (Lectures V-IX)

1) [Hints in Lecture V]

Rest-mass energy liberated by this reaction: E = m (^{12}C) + m(H) - m(^{13}C)

E = (12 + 1.0081) - 13.003355 = 4.745×10^{-3}

Fraction of rest-mass energy liberated per consumed nucleus of ^{12}C = 4.745×10^{-3} /12 = 3.95×10^{-4}

This means that the destruction of 1 nucleus of ^{12}C liberates 3.95×10^{-4} of the total restmass of ^{12}C burned in the star.

Total mass of ^{12}C in the star = 3×10^{-3} x Mstar = $3 \times 10^{-3} \times 2 \times 10^{30} = 6 \times 10^{27}$ kg

10% of the total mass of ^{12}C is consumed = $0.10 \times 6 \times 10^{37} = 6\ 10^{26}$ kg

⇒ total energy liberated E = fraction of rest mass x total mass of ^{12}C burned x c^2

= $3.95 \times 10^{-4} \times 6 \times 10^{26} \times (3 \times 10^{8})^2 \approx 2.1 \times 10^{40}$ kg m^2 s^{-2}

Given that 1J=1kg m^2 s^{-2} ⇒ total energy liberated E = 2.1×10^{40} J

Lifetime $\tau = E/L \approx 2.1 \times 10^{40} / (4\ 10^{26}) \approx 5.3 \times 10^{13}$ s $\approx 1.7\ 10^{6}$ yr

2) [See notes in Lecture V]

Convection in the stellar envelope is responsible for the transport of elements from the central regions where they are produced to the surface. The existence of this convective envelope is due to the increase of the opacities (peak around a temperature of ~ 10^4 K) as the effective temperature of the star decreases during its evolution.

Examples of such elements: He, produced in H burning regions (via the PP chains or the CNO cycle); ^{14}N produced produced by the CNO cycle (H burning); or C and O produced in He burning regions

- Kippenhahn diagram see Lecture V
- Ignition of H fusion reactions: all stars with $M > 0.07\ M_\odot$
- Ignition of C fusion reactions: all stars with $M \gtrsim 8\ M_\odot$

3) Deuterium: $T \approx 10^6$ K ; lithium: $T \approx 2 \times 10^6$ K; beryllium: $T \approx 3 \times 10^6$ K **[See notes in Lecture V]**

- CNO dominant at $T \approx 18\ 10^6$ K **[See Notes in Lecture V]**
- Because their central regions become degenerate before the temperature reaches the ignition temperature for He burning reactions ($T \approx 10^8$ K) **[Hints in lecture IV and V]**

4) - $P = K \rho^{1+1/n}$

- n=3/2 describes the structure of a fully convective object composed of an ideal gas or of a fully degenerate non-relativistic object
- Very low mass stars and brown dwarfs since they are fully convective ; white dwarfs or neutron stars (fully degenerate)

[Hints in lecture VII]

5) **[see notes in lecture VIII in section “Stability Considerations”]**