Lecture 3 : Stellar Remnants

SUMMARY

We describe the three types of remnants that a star can leave at the end of its life:

White dwarfs are inert cores of helium, carbon or oxygen, supported by *degenerate electron pressur*e.

Neutron stars are supported by degenerate *neutron* pressure. Observed as *pulsars,* as they send spinning beans of non-thermal electromagnetic radiation.

Black holes are often surrounded by hot *accretion discs, which shine brightly in X-rays and gamma-rays.*

The centre of many galaxies harbour a **super-massive black hole,** which can power an *active galactic nucleus.*

White dwarfs

A white dwarf is the inert core of a Sun-like star, after nuclear reactions have run out of fuel and the envelope has been blown out. A white dwarf composed either of helium if the star was not heavy enough to reach the helium-fusion stage, or of the main products of helium fusion, carbon and oxygen. Although the mass of a white dwarf is between 0.5 and 1.4 Solar Masses, its size is comparable to that of the Earth.

White dwarf have no internal source of energy, and therefore slowly cool down with time. How long does it take ? This is the ``Kelvin-Helmolz'' timescale that we have encountered before, when discussing the source of energy of the Sun. It would take a few million years for the Sun to loose all the energy from gravitational contraction, were it not for nuclear fusion. Since white dwarfs has similar masses, similar temperatures but only a tiny fraction of the surface (~1/10'000), their Kelvin-Helmolz timescale is ~10'000 longer than that of the Sun, $\sim 10^7 \cdot 10^4 = 10^{11}$ years. It therefore takes longer than the present age of the Universe ($\sim 10^{10}$ years) for a white dwarf to loose its internal heat. As a result, even the oldest white dwarfs are still visible as faint source of light.

Why is a white dwarf so small? When nuclear reactions cease, nothing prevents the core of the star to contract under the pull of its own gravity, until electrons in the gas become so tightly packed that they would violate the rules of quantum mechanics if they were to move closer to each other. This special source of resistance against further compression is called *electron degeneracy pressure.*

Degenerate electron pressure

Degenerate pressure is a fully quantum phenomenon, without any classical equivalent. Degenerage pressure is a consequence of two fundamental principles of quantum physics:

- the Exclusion Principle *(Pauli)*
- The Uncertainty Principle *(Heisenberg)*

These two principles explain why “matter takes up space”, even though elementary particles are point-like entities with no intrinsic spatial extension.

In quantum physics, particles of matter like electrons, protons and neutrons are *fermions*. The ***exclusion principle*** states that two fermions cannot occupy the same quantum state. The other kind of particles, called *bosons,* are not subjected to the exclusion principle. Photons, for instance, are bosons. Light rays can cross each other in infinite numbers, but it is not possible to stack electrons, protons and neutrons closer. This corresponds to the fundamental difference in our intuition between “matter” and things like light, magnetism and other “spooky” features of the natural world. Matter “takes up space’’, while light doesn’t.

Fermions and Bosons

The two types of particles are named after the physicists Fermi and Bose. The rule is that particles with integer *spin (0,1,2...),* like the photon, are bosons, while particles with half-integer spin (1/2, 3/2, ...) are fermions. The spin is a quantum number that describe the intrinsic angular momentum of a particle. Particles with integer spin add up when occupying the same state, so that one photon plus one photon gives two photons. Wavefunctions with half-integer spin, however, cancel out, so that one electron plus another electron in the same quantum state give no electron.

The ***uncertainty principle*** states that the product of the position and momentum uncertainty of an elementary particle cannot be smaller than the Plank constant.

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

As a consequence, an electron with a given energy cannot be confined to a volume of space smaller than the Plank constant divided by that energy.

Taken together, these two principles imply that a collection of fermions cannot be compressed into a smaller region of space than a certain limit. The uncertainty principle sets the minimum amount of space occupied by a single fermion with a given momentum, and the exclusion principle ensures that other fermions will have to find other positions.

In a white dwarf, electrons are compressed to the point that they will fill all available position/momentum combinations starting from the lowest momentum possible upwards. They will occupy a certain volume, even without any contribution from the thermal energy. In that case the electrons are called *degenerate.*

We can estimate the size of a white dwarf by using the two following assumptions:
(1) the electrons occupy the smallest amount of space compatible with quantum physics
(2) their pressure must be sufficient to balance the pull of gravity.

Another way of stating (2) is to say that the mean kinetic energy of the electrons equals the mean gravitational potential energy. I.e.

$E_k = E_g$

The gravitational potential energy per unit mass is

$Eg \simeq GM / R$

The kinetic energy is

$Ek \simeq n\, p^2 / m_e$

where n is the number of electrons per unit mass. We will assume that each electron occupies a region of space Δx and has a momentum Δp with $\Delta x\, \Delta p = \hbar/2$.

Solving the three equations above for R yields the mass-radius relation of an electron-degenerate stellar body:

$$\mathbf{R} \simeq n^{5/3}\, \hbar^2 / (2m_e\, G\, \mathbf{M}^{1/3})$$

The size of a carbon/oxygen solar-mass white dwarf is comparable to that of the Earth.

Electron vs neutron degeneracy

In the uncertainty principle, the phase space is defined in position, Δx, and momentum, Δp. Because $p=m\,v$ *(definition of momentum),* a more massive particle will be able to satisfy the uncertainty principle at the same value of momentum with a correspondingly smaller values of Δx.

A neutron is ~1700 times heavier than an electron. A neutron star, supported by neutron degeneracy pressure, is ~10 km in radius. A white dwarf, supported by electron degeneracy pressure, is about the size of the Earth.

One remarkable feature of this mass-radius relation is that, in contrast to normal stars, as the mass gets higher, the radius get *smaller.* As more mass is added, the momentum of the electron must increase to compensate for the increase gravity, which allows the Δx to decrease. The result is a net decrease in volume.

N.B. in normal stars, the gas is hot enough to provide the pressure to support the gravity with normal gas pressure (due to the shock of particles against each other), the electrons are not degenerate.

Core collapse

At some point, the pressure needed for the electrons to balance the force of gravity will require their velocity to reach the velocity of light, i.e. $p = m_e c$. Since electrons cannot move faster than the speed of light, beyond that point, electron degeneracy pressure is no longer sufficient to counter gravity. This sets an upper limit for the mass of a white dwarf. This limit is called the Chandrasekhar limit, and amounts to about 1.4 Solar Masses.

What happens if a stellar remnant is heavier than this? Electrons give way. This is a runaway process, since as the stellar core contracts, the gravity gets stronger. The result is a supernova explosion.

What happens next to the stellar core depends on its total mass. Below about 3 solar masses, the electrons combine with protons in the atomic nuclei to form neutrons, which provide another barrier against gravity. The result is a neutron star.

At higher core masses, even neutron degeneracy pressure is not sufficient to counterbalance gravity, and the core collapses into a black hole.

Neutron star

A neutron star is supported by neutron degeneracy pressure just as a white dwarf is supported by electron degeneracy pressure. Repeating the calculation above, we can guess that the ratio of the size of a neutron star to a white dwarf will be that of the mass of the neutron to the mass of the electron. Indeed, neutrons are about 1800 time more massive than electrons, and a neutron star is only about 10 km in diameter.

Neutron stars are strong contenders for the title of strangest objects known to astrophysics. They are fully quantum systems, a single superfluid, superconducting entanglement of neutrons kept separate only by the exclusion principle[1]. Because of the ``spinning skater'' effect, neutron stars rotate much faster than Sun-like stars (they keep the same angular momentum, but gathered on a $\sim 10^4$ times more compact object). Neutron stars rotate in seconds or less, some of them spin entirely every thousandth of a second. Neutrons have no electric charge, but a small magnetic moment[2]. In a neutron star, these tend to be all aligned with each other, adding up to generate a huge magnetic field. The strength of the magnetic field means that escaping particles can only move along very narrowly defined channels around the star.

Pulsars

So, how do these stars look like from a distance? Like radio beacons. For an observer placed in the right direction relative to their magnetic field, neutron stars send a strong

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[1] at least, in our current understanding. Depending on the properties of quarks, the inside of neutron stars could be stranger still.

[2] quantum mechanics again: although neutrons have no net charge, they are composed of three quarks with some charge. The sum of charges is zero, but the presence of the charges produces a magnetic field.

pulse of radio waves, each time the strong beam of radiation generated by the electrons trapped by their magnetic field crosses the line of sight (see *non-thermal radiation*), just like a lighthouse in the night.

These sources of very regular radio pulses in the sky are called *pulsars*. The fist pulsar was detected in 1967.

Non-thermal radiation

Not all sources of radiation follow Plank's law (blackbody radiation emitted by a surface in temperature equilibrium). There are other ways to generate radiation, that produce very different spectral distributions. Two examples relevant in astrophysics are:

Synchrotron emission: produced by charged particles spiralling around magnetic field lines .

Bremsstrahlung radiation: produced by the acceleration of free electrons after interaction with positively charged hydrogen ions.

The radiation emitted by neutron stars consists mainly of these types of non-thermal emission, which can produce strong fluxes in radio waves and X-rays without a corresponding emission in visible light.

Supernovae

How much energy is liberated when the core of a star collapses from electron degeneracy to neutron degeneracy? We can calculate the difference of gravitational potential energy between a star of 1.4 Solar Masses with the size of a white dwarf, and one with the size of a neutron star:

$$\Delta E \sim -\left(\frac{GM^2}{r_{WD}} - \frac{GM^2}{r_{NS}}\right)$$

Since the first term is much smaller than the second:

$$=> \Delta E \sim \frac{GM^2}{r_{NS}}$$

For a neutron star radius of 10 km, $\Delta E \sim 10^{46} J$.
The result is much larger than the luminosity of the Sun ($\sim 10^{26}$ W) integrated over its whole lifetime ($\sim 10^{10}$ yrs $\sim 10^{17}$ s)!

The inner core collapse is practically instantaneous, but since the explosion blows out the outer parts of the star, a supernova is visible for several days after its explosion. Even spread over several days, the luminosity of a supernova is comparable to that of a whole galaxy. Supernovae can be detected at very large distances in the Universe. They are used as beacons to measure the distance of remote galaxies.

A supernova explosion leaves behind expanding shells of hot gas. The crab nebula, for instance, is the remnant of a supernova that exploded in the year 1054 (or, more accurately, the light from its explosion reached the Earth in 1054; since the crab nebula is about 6500 light-years away, the actual moment of the explosion was some time in the fifth millennium B.C.).

The brightest supernova visible from Earth in recent history is SN1987A, which exploded in the Large Magellanic Cloud, a small satellite galaxy of the Milky Way.

The formation of a neutron star requires the transformation of many protons into neutrons. As we have seen from the nuclear reaction inside the Sun, neutrinos need to be produced in this process to conserve the number of leptons (electron-like particles). In parallel to the visible explosion, supernovae therefore send out a short but very intense pulse of neutrinos. In fact, most of the energy of the explosion is carried away by neutrinos, with only a small fraction left to photons. The neutrinos from SN1987A were even detected by underground solar neutrino detectors (3 hours *before* the supernova was visible in the sky).

Black holes

Stellar cores heavier than about 3 Solar Masses run into a new type of problem when their core collapses. At the size of a neutron star, the escape velocity at their surface is higher than the speed of light. In other words, even a particle moving outwards at the speed of light will be drawn back into the core. As a consequence, the stellar core collapse into a black hole.

Let us calculate the size of a black hole of 1 Solar Mass by equating the escape velocity to the speed of light. The escape velocity from a body of mass M and radius r is:

$$v^2 = 2\,G\,M\,/\,r.$$

(This is found be requiring the sum of the kinetic energy $\frac{1}{2}mv^2$ and the potential energy at the surface, $-GMm/r$ to be zero, so that escape to infinity is allowed).

The escape velocity is equal to the speed of light for:

$$r = 2\,G\,M\,/\,c^2$$

A rigourous derivation, using general relativity, gives the same answer. It was first done by Karl Schwarzschild (1873-1916), and the corresponding radius is known as the *Schwarzschild radius*.

For 1 solar mass, the Schwarzschild radius is about 3 km.

The physics of black holes derives primarily from the theory of general relativity, that describes how space-time reacts to the presence of matter and energy. There are planty of fascinating things to say about the mathematics and physics of black holes. From the point of view of astrophysics, however, most of it is of little direct relevance, since it cannot be observed. For all practical purposes, a black hole can be considered as a certain amount of mass concentrated in a negligible volume and producing no radiation.

Observing black holes

Just as it would have been difficult to predict that neutron stars would manifest themselves are regular radio pulses, it was not obvious a priori what observable phenomenon would reveal the presence of black holes.

A black whole in isolation is expected to be dark, as the name implies. However, if the black hole is accreting matter from some source, it may become detectable. Let us consider a disc of gas in orbit around a black hole. Since the orbital velocity depends on the distance, the disc will suffer some amount of shear, which will cause friction. The gas will heat up, and since the energy is extracted from the orbital motion, it will spiral inwards - until it feeds the black hole. As we get closer to the black hole, the orbital velocities will become enormous (a significant fraction of the speed of light). The gas will be heated to very high temperature. For a stellar mass black holes, the temperature of the gas will reach millions of degrees. Using Wien's law for the blackbody radiation, we expect the resulting thermal radiation to be emitting at wavelengths a thousand times shorter than visible light, which is the domain of X-rays.

It is as intense sources of X-rays that black holes make themselves known to us. For instance, in the Cygnus X-1 binary system, a normal star orbits every 5.6 days around an invisible companion of 8.7 Solar masses. The system is a large source of thermal X-rays, indicating the presence of a disc of very hot gas. The star looses part of its envelope to a very hot accretion disc spiralling towards a compact and invisible object.

Many such X-ray binaries are known, and the evidence for stellar-mass black holes is now compelling. Isolated black holes must also be common, but as long as no matter is fed onto them, they remain dark.

The luminosity of a black hole

The luminosity of a black hole - or, more accurately, of the accretion disc around a black hole - is powered by the gravitational potential energy of the matter falling into it.

$$\Delta E \sim \frac{GM\Delta m}{r}$$

$$L_{acc} = \frac{\Delta E}{\Delta t} \sim \frac{GM\Delta m}{r\Delta t}$$

$$=> L_{acc} \sim \frac{GM\dot{m}}{r}$$

Because of the small size and large mass of a black hole, the energy radiated is very large even for small values of the mass accretion rate. For instance, 1 kg of mass falling into a solar-mass black hole can produce an accretion luminosity of $7 \cdot 10^{-11} \cdot 2 \cdot 10^{30}$ / 3000 $\approx 3 \cdot 10^{16}$ J, which is similar to what would be produced by the entire conversion of 1 kg of matter to energy ($mc^2 \sim 10^{17}$ J).

Galactic black holes

Very hot accretion discs are also observed at the centre of some galaxies. From the features of their X-ray (and gamma-rays) emissions, the same type of inference can be made: the brightness and temperature of the accretion disc can be used to constrain the mass and size of the unseen accreting object.

For instance, in the galaxy NGC 4261, an accretion disc surround an object with 1.2 10^9 Solar Masses, confined within the size of the Solar System. Modern physics offers no plausible alternative to the explanation of these observations in terms of a black hole of more than one billion solar masses sitting at the centre of that galaxy.

Quasars and active galactic nuclei

Galactic-scale black holes also provide an explanation for another type of phenomenon: that of *quasars* and active galactic nuclei (“AGN”).

Many galaxies harbour a very intense and active source of high-energy radiation at their centre. Sometimes the source is so intense that the rest of the galaxy is difficult to detect. The “quasars” (=quasi-stellar source) are such objects. They are intense sources of UV, X-rays and gamma-rays, situated at huge distances (several billion light-years) and with no obvious associated galaxy. Although the first quasar was characterised in the 1960s, only in the 1990s did it become clear that all quasars were associated with the nucleus of a galaxy.

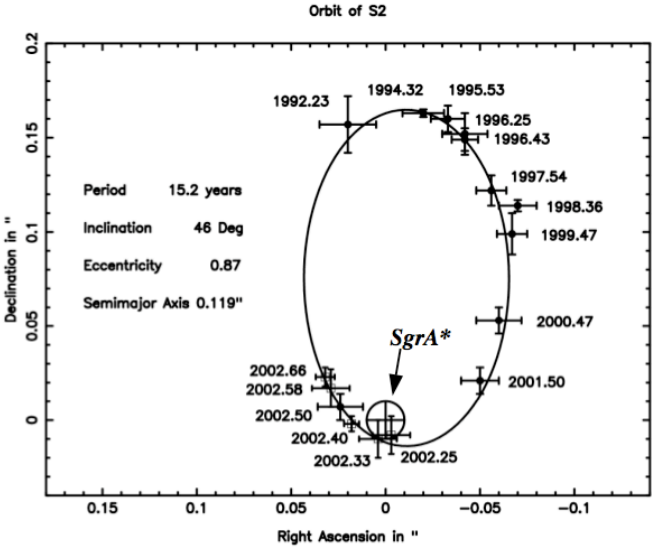
Quasars are now considered as a type of *active galactic nuclei (AGN).* Quasars and AGN can be explained by the presence of compact accretion discs around very massive objects, which are believed to be galactic-scale black holes.

AGNs are the most luminous objects in the Universe, and the brightest can be detected from any points in the visible Universe. For this reason, they have been used in the disc send with the Voyager mission to explain our position to potential cosmic interlocutors. The position of the Earth was given relative to a set of bright quasars.

For the same reason, AGN were also used to probe the topology of the Universe, i.e. to explore the hypothesis that the Universe is looping on itself, so that the same regions of space would be visible in more than one direction at once. Although such studies are made more complicated by the fact that galaxies evolve over time, and that the look-back time would be different for different directions, it now seems established that the Universe is not looping on any scale smaller than the entire visible cosmos.

The central black hole of the Milky Way

Observations of the motion of stars close to the centre of our own galaxy have revealed the presence of a compact and invisible object, with a mass of about 4 millions times the mass of the Sun. A super-massive black hole seems to sit at the centre of the Milky Way. It is much less active than an AGN. Still, its presence does suggest that most galaxies may harbour a giant black hole in their centre.



Orbit of one star around the galactic centre ("SgrA*"). The orbital period is 15.2 years, and the semi-major axis is 0.119 arcsec, which at the distance of the galactic centre corresponds to ~1000 AU. Applying Kepler's third law gives a mass of $4 \cdot 10^6$ M_{Sun} for the central mass.