Lecture 2 Summary

* Classical free electron gas Model – metals
  - Drude
  - electrons behave as classical gas
  - independent electron approx. } assume
  - free electron approx. } $U = 0$

- resistance comes from collisions of electrons with ion cores $\rightarrow V_d,\ \tau$

$$V_d = \frac{-eE\tau}{m} \quad \text{and} \quad J = -nev_d$$

$$\rightarrow J = \underbrace{\left(\frac{ne^2\tau}{m}\right)}_{\sigma} E \quad \text{ohms law ✓}$$

$$J = \frac{I}{A} = \frac{1}{A}\frac{\Delta Q}{\Delta t} = \frac{1}{A}\frac{(-ne)(AL)}{L/v} = -nev$$

But heat capacity $\quad C = \frac{dE}{dT} = \frac{d}{dT} 3N_A k_B T = \text{constant}$ ✗

and Wiedemann–Franz law $\quad \frac{K}{\sigma T} = \text{constant}$ ✓ (not quite correct const.)

use classical gas: $K = \frac{1}{3} v^2 \tau C$; $C = \frac{3}{2} N k_B$

* Quantum version of Drude (FEG) model is better

Solve single e^- Schrodinger Eqn with $U = 0$

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\underline{r}) + U\psi(\underline{r}) = E\psi(\underline{r})$$

gives $\nabla^2\psi(\underline{r}) = -k^2\psi(r) \qquad k^2 = \frac{2mE}{\hbar^2}$

soln's $\psi = A \exp i(\underline{k}\cdot\underline{r} - \omega t)$ ie plane waves

$E = \frac{\hbar^2 k^2}{2m}$ ie parabolic "bands"

By employing periodic boundary conditions, we define the set of allowed k's

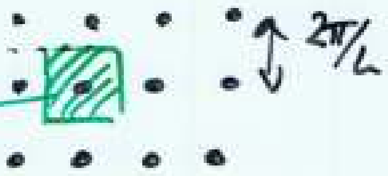
ie $\psi(x+L) = \psi(x)$ in one-dimension

circum. = L

in 3D ... a cube of side length L.

$$\rightarrow k_x = \frac{2\pi n_x}{L} \quad k_y = \frac{2\pi n_y}{L} \quad k_z = \frac{2\pi n_z}{L} \qquad n_x, n_y, n_z = 0, \pm 1, \pm 2 \ldots$$

$\therefore$ in recip. space (2D)



* Density of Quantum/electron states, $G(k)$ $[G(\varepsilon)]$

$= \frac{dN}{d\varepsilon}$ ① Consider a spherical shell of radius k and thickness dk ... it has Vol = $4\pi k^2 dk$

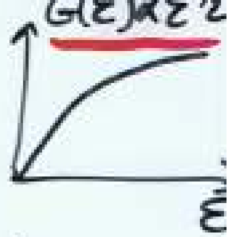
② Vol. occupied by 1 -state = $2\pi/L^3$ (in 3D)

③ 2 electrons per quantum state

$$\therefore \; G(k)dk = 2 \times \frac{4\pi k^2 dk}{2\pi/L^3}$$

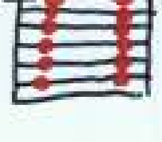
Remember $k = \frac{\sqrt{2m\varepsilon}}{\hbar}$ $\therefore$ $dk = \frac{1}{\hbar}\frac{m}{\sqrt{2\varepsilon}}d\varepsilon$

$$\rightarrow G(\varepsilon) = \frac{dN_{el}}{d\varepsilon} = G(k)\frac{dk}{d\varepsilon} = \frac{V}{2\pi^2}\frac{(2m)^{3/2}}{\hbar^3}\varepsilon^{1/2}$$



Fermi Energy - the energy of the highest filled electron state at T = 0K.

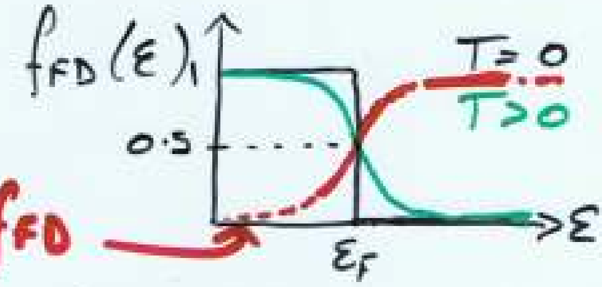
$$N_{el} = \int_0^{\varepsilon_F} G(\varepsilon)d\varepsilon$$



$G(\varepsilon)$ → how many (distribution) of q. states

$f(\varepsilon)$ → probability that a state is occupied

Fermi-Dirac distribution function $\left(f_{FD} = \frac{1}{\exp[(\varepsilon-\varepsilon_F)/k_BT]+1} \right.$



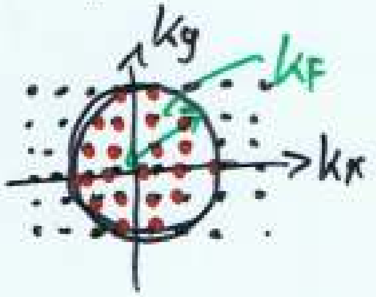
probability of a state being empty → $1-f_{FD}$

We generally ignore temp. dependence of $\varepsilon_F(\mu)$ since $\varepsilon_F \gg k_BT$.

$f_{FD} \to f_{MB}$ when $\varepsilon - \varepsilon_F \gg k_BT$

$$f(\varepsilon) \approx {}^{-\left(\frac{\varepsilon-\varepsilon_F}{k_BT}\right)}$$

Fermi wavevector



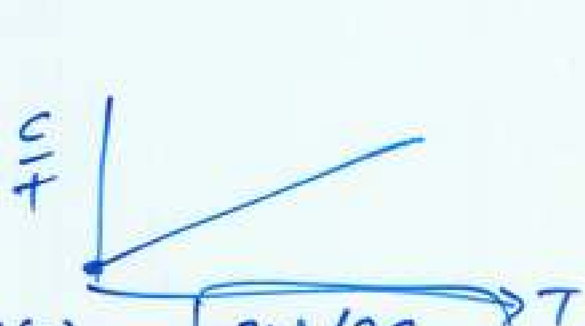
FEG: isotropic

~ 10 nm⁻¹

Fermi velocity $v_F \sim 10^6\,\mathrm{ms^{-1}}$ (ie e^- still moving at T=0K!)

Fermi temp $T_F \sim 10^5\,\mathrm{K}$

$\varepsilon_F \sim 2 \to 10\,\mathrm{eV}$.



$$\frac{C}{T} = \frac{G(\varepsilon)}{T} = \boxed{\frac{3N/2\varepsilon_F}{T}}$$