

## Absorption coefficient

$$\alpha(h\nu) = A \sum_{i,f} f_{if} g_i g_f$$

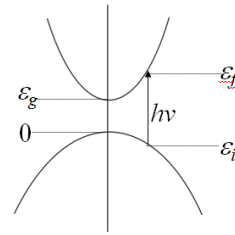
Summed for all possible transitions between states separated by an energy difference of  $h\nu$ .

probability of transition from the initial to the final state

density of electrons in initial state

density of available (empty) final states

- We will consider absorption transitions between 2 direct valleys assuming all valence states are filled, and all conduction states empty (true for undoped semiconductors at  $T = 0\text{K}$ ), and all momentum-conserving transitions are allowed (*i.e.* probability function,  $f$  is independent of energy)
- Photon energy must be equal to the energy gap, or larger, *i.e.*  $\nu \geq \varepsilon_g/h$ 
  - Absorption edge given by  $\nu_0 \geq \varepsilon_g/h$
- Momentum of photon ( $h/\lambda \ll$  crystal momentum ( $h/a$ ) therefore the excited electron state should have essentially the same  $k$ -value as the initial state.
  - a photon is annihilated and an electron-hole pair created.
  - Energy and momentum are conserved and electron and hole have opposite  $k$  (slide 6.5) (neglecting very small momentum of light)

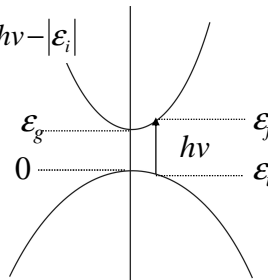


$\varepsilon_f$ : final energy state (note: not  $\varepsilon_r$  - Fermi energy level)  
 $\varepsilon_i$ : initial energy state



## Allowed direct transitions: Absorption

- i.e.* every initial state is connected with a final state:  $\varepsilon_f = h\nu - |\varepsilon_i|$
- In parabolic bands:  $\varepsilon_f - \varepsilon_g = \frac{\hbar^2 k^2}{2m_e^*}$  and  $\varepsilon_i = \frac{\hbar^2 k^2}{2m_h^*}$
- Therefore  $h\nu - \varepsilon_g = \frac{\hbar^2 k^2}{2} \left( \frac{1}{m_e^*} + \frac{1}{m_h^*} \right)$   
 $= \frac{\hbar^2 k^2}{2} \left( \frac{1}{m_r} \right)$  where  $m_r$  is reduced mass,  $1/m_r = 1/m_e^* + 1/m_h^*$
- Density of directly associated states,  $g(h\nu) = \frac{4\pi(2m_r)^{3/2}}{h^3} (h\nu - \varepsilon_g)^{1/2}$  (slide 2.6)
- Hence  $\alpha(h\nu) = A^* (h\nu - \varepsilon_g)^{1/2}$
- Summary:** Photon ensures energy is conserved;  $k_f = k_i$



For an index of refraction of 4, and assuming  $m_e^* = m_h^* = m_0$ , then  
 $\alpha(h\nu) \approx 2 \times 10^4 (h\nu - \varepsilon_g)^{1/2} \text{ cm}^{-1}$   
 (with  $h\nu$  and  $\varepsilon_g$  in eV)

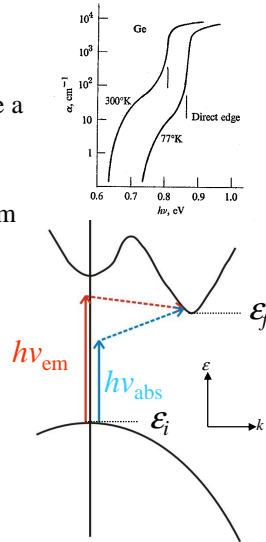


## Indirect transitions

- Transition between indirect valleys (e.g. in Ge)
- Photon cannot provide a change in momentum, therefore a two-step process is required.
- Momentum conserved via a **phonon** interaction (quantum lattice vibration)
- A phonon (energy  $\epsilon_p$ ) is either **emitted** or **absorbed**:

$$h\nu_{abs} = \epsilon_f - \epsilon_i - \epsilon_p \quad 1.$$

$$h\nu_{em} = \epsilon_f - \epsilon_i + \epsilon_p \quad 2.$$



## Indirect transitions

- All occupied states of valence band can connect to all empty states of conduction band.

- Density of initial states, 
$$g(\epsilon_i) = \frac{4\pi(2m_h^*)^{3/2}}{h^3} |\epsilon_i|^{1/2} \quad 3.$$

- Density of final states, 
$$g(\epsilon_f) = \frac{4\pi(2m_e^*)^{3/2}}{h^3} (\epsilon_f - \epsilon_g)^{1/2} \quad 4.$$

- Substitute equations 1. and 2. on previous slide in equation 4:

$$g(\epsilon_f) = \frac{4\pi(2m_e^*)^{3/2}}{h^3} (h\nu - \epsilon_g \mp \epsilon_p + \epsilon_i)^{1/2} \quad 5.$$

- For indirect transitions, absorption coefficient  $\alpha$  is proportional to product of equations 3 and 5, integrated over all possible combinations of states separated by  $(h\nu \pm \epsilon_p)$

## Indirect transitions

- BUT**,  $\alpha$  is also proportional to probability of interaction with phonons, which itself is a function  $f(n_p)$  of the number of phonons. Phonons obey Bose-Einstein statistics...

$$n_p(\epsilon_p) = \frac{1}{\exp(\epsilon_p/k_B T) - 1} \quad \begin{array}{l} \text{(\# of phonons} \\ \text{of energy } \epsilon_p) \end{array}$$

- Hence  $\alpha(h\nu) = A f(n_p) \int_0^{(h\nu - \epsilon_g \mp \epsilon_p)} |\epsilon_i|^{1/2} (h\nu - \epsilon_g \mp \epsilon_p + \epsilon_i)^{1/2} d\epsilon_i$

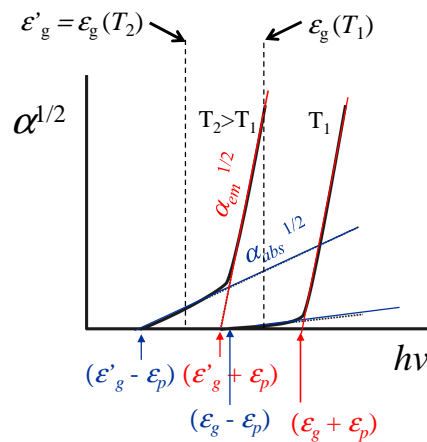
- After integration and substitution of the Bose-Einstein function:

$$\alpha_{abs}(h\nu) = \frac{A(h\nu - \epsilon_g + \epsilon_p)^2}{\exp(\epsilon_p/k_B T) - 1} \quad \text{phonon absorption}$$

$$\alpha_{em}(h\nu) = \frac{A(h\nu - \epsilon_g - \epsilon_p)^2}{1 - \exp(-\epsilon_p/k_B T)} \quad \begin{array}{l} \text{phonon emission} \\ \text{(note: different factor on bottom} \\ \text{since probability of phonon} \\ \text{emission is proportional to } n_{ph} + 1) \end{array}$$



## Indirect transitions



Both transitions are possible when  $h\nu > \epsilon_g + \epsilon_p$ .

But phonon concentration is very small at low temperatures – therefore  $\alpha_{abs}$  is also small

Note:  $\epsilon_g$  shifted with temperature to reflect some temperature dependence of energy gap.

*Explanation:* As temperature increases, the band gap energy decreases because the crystal lattice expands and the interatomic bonds are weakened. Weaker bonds means less energy is needed to break a bond and get an electron in the conduction band.

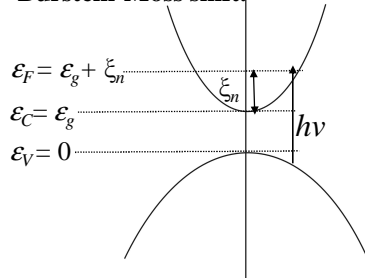


## Burstein-Moss shift.

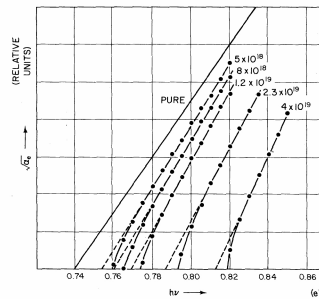
See slide 10.9

- If semiconductor is **degenerately** (heavily) doped, the Fermi level can be *inside* the band (*i.e.* conduction band for n-type semiconductor) by a quantity  $\xi_n$ .
- Since states between  $\epsilon_C$  and  $\epsilon_F$  are already filled, fundamental transitions to states below  $\epsilon_g + \xi_n$  are forbidden.
- Hence absorption edge shifts to higher energies:

### Burstein-Moss shift.



→  
 Ge at 4.2K (*i.e.* only phonon emission possible): absorption coefficient as a function of doping concentration.

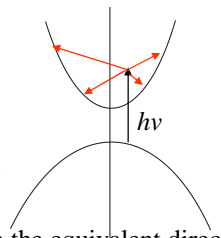


## Indirect transitions

- In *heavily doped indirect* band-gap semiconductors, it is also possible to conserve momentum by a scattering process without interaction with a phonon, via
  - electron-electron scattering
  - impurity scattering
- Now the scattering probability is proportional to the number, of scatterers,  $N_{sc}$ :

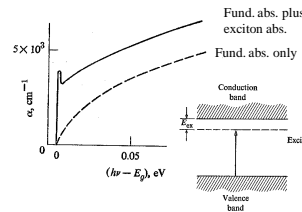
$$\alpha(h\nu) = AN_{sc}(h\nu - \epsilon_g - \xi_n)^2$$

- *Indirect transitions between direct valleys* are also possible.
- Momentum conserved by 2<sup>nd</sup> order process (phonons, or scattering)
- Absorption coefficient same as previous indirect transitions.
- Multistep processes will have a much lower probability than the equivalent direct transition.



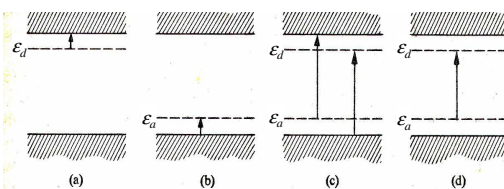
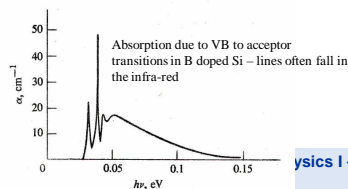
# Absorption

- In discussing fundamental absorption, we've assumed that the electron and hole created are each "free". However they will attract one another and may form a bound state, in which they revolve around their combined centre of mass. This state is called an exciton.
- Its binding energy is very small  $\sim 0.01\text{eV}$  (i.e. just below the CB edge)



- (a) neutral donor accepts photon and electron excited into CB (or higher impurity level)
- (b) analogous to (a) - VB to neutral acceptor
- (c) VB to ionized donor, or from ionized acceptor to CB.
- (d) Ionized acceptor to ionized donor.

## Impurity absorption (no creation of e-h pair)

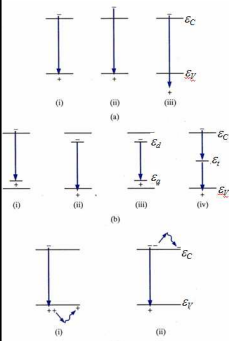
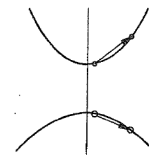


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Slide 14-9

# Free carrier absorption

- Absorption of radiation by electrons or holes to change the state in which they exist, but without becoming excited into other bands
- These are labelled intraband transitions, and they dominate the absorption spectrum below the fundamental edge.



## Recombination of electron-hole pairs and photon emission

### (a) basic interband transitions

- (i) intrinsic emission very close to bandgap energy
- (ii) and (iii) energetic electrons or holes – energy of emitted photon  $>$  bandgap energy. There will then be an emission spectrum and a associated bandwidth.

### (b) involving impurity states

- (i) CB-acceptor; (ii) donor-VB; (iii) donor-acceptor; (iv) "deep trap" captures electrons and holes with equal probability (non-radiative)

- (c) **Auger recombination processes** (3-particle)– non radiative – occur in heavily doped materials – energy transferred to other charge carriers and eventually lost as heat

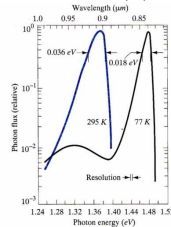


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## Recombination and photon emission

- Processes shown in (a) on last slide indicate that the emission of a photon is not necessarily at a single, discrete energy.
- Spontaneous emission rate has the form  $I(\nu) \propto \nu^2 (h\nu - \epsilon_g)^{1/2} \exp\left[\frac{-(h\nu - \epsilon_g)}{k_B T}\right]$



Peak photon energy decreases with temperature because bandgap energy decreases with temperature.

- Emission spectra of GaAs:
- However, not all recombination processes are radiative – but an efficient luminescent material is one where radiative transitions dominate.
- Quantum efficiency defined as ratio of radiative recombination rate to the total recombination rate for all processes *i.e.*

$$\eta = R_r / (R_r + R_{nr}) = R_r / R$$

## Luminescent efficiency

- We define the radiative and nonradiative recombination rates as

$$R_r = \frac{N}{\tau_r}; \quad R_{nr} = \frac{N}{\tau_{nr}} \quad (N \text{ is \# of electron-hole pairs})$$

- Recombination rate is inversely proportional to lifetime, then

$$\eta = \frac{\text{radiative recombination lifetime} \rightarrow \frac{1}{\tau_r}}{\text{nonradiative recombination lifetime} \rightarrow \frac{1}{\tau_{nr}} + \frac{1}{\tau_r}} = \frac{1/\tau_r}{(\tau_r + \tau_{nr})/\tau_r \tau_{nr}} = \frac{\tau_{nr}}{\tau_{nr} + \tau_r}$$

- i.e.* For a high luminescent efficiency, nonradiative lifetime must be large
- Interband recombination rate of electrons and holes is directly proportional to number of electrons available and number of empty states:  $R = Bnp$
- here  $B$  constant of proportionality.  $B$  for indirect bandgap materials is  $\sim 10^6$  times smaller than that of direct gap materials!
- One problem with emission of photons from direct bandgap materials is **reabsorption** of emitted photons!

$$\frac{dN}{dt} = G - R \quad \rightarrow \quad \text{steady state solution, } dN/dt = 0 \rightarrow N_0 = G\tau$$