Statistics of donors and acceptors

- **Electron concentration in the donor states**, $n_d = N_d \times f_d(\varepsilon)$, where N_d is the concentration of donor atoms, and the probability distribution function based on Fermi-Dirac (slide 2.7) is

$$f_d(\varepsilon) = \frac{1}{1+\frac{1}{2}\exp\left(\frac{\varepsilon_d-\varepsilon_F}{k_B T}\right)}$$

- Notice the "new" factor of ½: Each donor level has two possible spin orientations for the donor electron, (*i.e.* each donor has two quantum states).
- However the insertion of an electron into one of the states satisfies the vacancy requirement of the atom, and the insertion of a second electron is forbidden.
- This factor is sometimes written as $1/g$, where g is the degeneracy factor.

see Neamen, p132 for full derivation

Statistics of donors and acceptors

- Hence, the concentration of electrons occupying the donor level

$$n_d = \frac{N_d}{1+\frac{1}{2}\exp\left(\frac{\varepsilon_d-\varepsilon_F}{k_B T}\right)} = N_d - N_d^+ \quad (1)$$

and similarly

$$p_a = \frac{N_a}{1+\frac{1}{4}\exp\left(\frac{\varepsilon_F-\varepsilon_a}{k_B T}\right)} = N_a - N_a^-$$

where N_d^+ is concentration of ionised donors, N_a is concentration of acceptor atoms, ε_a is acceptor energy level, p_a is concentration of holes in the acceptor states and N_a^- is the concentration of ionised acceptors.

- *i.e.* a hole in the acceptor state corresponds to an acceptor atom that is neutrally charged and still has an "empty" bonding position.
- Note the factor of ¼ in expression for p_a (applying to Si and GaAs) - due to complex valence band structure (slide 7.7)

Complete Ionisation

- If $(\varepsilon_d - \varepsilon_F) >> k_BT$, then (*i.e.* for non-degenerate semiconductors we can use the MB approx.)

$$n_d \approx \frac{N_d}{\frac{1}{2}\exp\left(\frac{\varepsilon_d - \varepsilon_F}{k_BT}\right)} = 2N_d \exp\left[\frac{-(\varepsilon_d - \varepsilon_F)}{k_BT}\right]$$

Slide 2.8

- The Boltzmann approximation is also valid for the electrons in the conduction band, so that

$$n_0 = N_C \exp\left[\frac{-(\varepsilon_C - \varepsilon_F)}{k_BT}\right]$$

- Hence

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_C}{2N_d}\exp\left[\frac{-(\varepsilon_C - \varepsilon_d)}{k_BT}\right]}$$

is the relative number of electrons in the donor state compared to the total number of electrons – independent of ε_F.

ionisation energy of the donor electrons

Complete Ionisation and Freeze-out

- *e.g.* (check this!) for Phosphorus doping ($N_d = 10^{16}$ cm^{-3}) in silicon at 300K , there are very few electrons (0.4%) in the donor state compared with the conduction band.

- *i.e.* THE DONOR STATES ARE ESSENTIALLY COMPLETELY IONISED AT ROOM TEMPERATURE and therefore almost all donor impurity atoms have donated an electron to the conduction band.

- Similarly, at room temperature, there is complete ionisation of acceptor atoms (*i.e.* each has accepted an electron from valence band, $p_a = 0$)

- At absolute zero, complete opposite happens: all electrons are in their lowest state *i.e.* for an *n*-type material, each donor state contains an electron

- So, $n_d = N_d$ or $N_d^+ = 0$, and hence (from eqn (1) on slide 11.2) exp $[(\varepsilon_d - \varepsilon_F)/k_BT]$ must equal 0.

$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{\varepsilon_d - \varepsilon_F}{k_BT}\right)}$$

Freeze-out

- Since T = 0K, this occurs in an n-type material when $[(\varepsilon_d - \varepsilon_F)/k_BT] = -\infty$ so that $\varepsilon_F > \varepsilon_d$.

- For p-types at T = 0K, the impurity atoms will not contain any electrons so the Fermi energy is below the acceptor energy level.

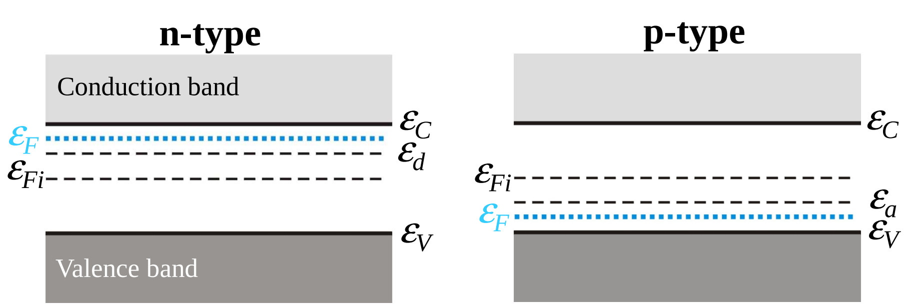
- A detailed analysis shows that at T = 0K, ε_F is positioned halfway between ε_C and ε_d for n-type, and halfway between ε_V and ε_a for p-type materials.

- **<u>Freeze-out</u>**: when no electrons from the donor state are thermally elevated into the conduction band **or** when no electrons from valence band are elevated into acceptor state.

- Between absolute zero and room temperature (complete ionisation) – partial ionisation of donor/acceptor atoms (later).

Freeze-out

Energy level diagrams at T = 0K:



Example: Determine the temperature at which 90% of acceptor atoms are ionised. Consider p-type silicon doped with boron at a concentration of $N_a = 10^{16}$ cm^{-3}.

Answer:

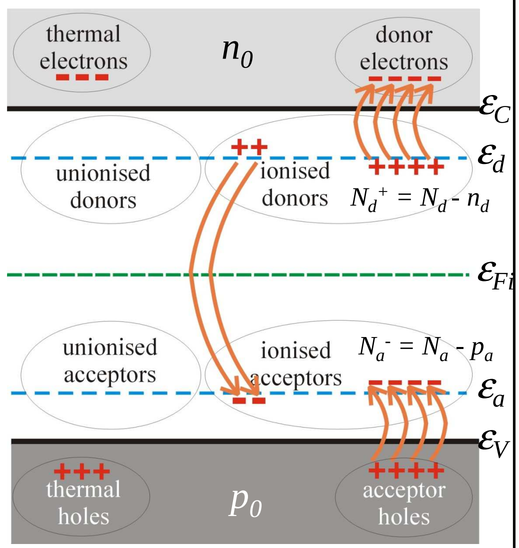
Compensated Semiconductors

- Contains both donor and acceptor impurities in same region *e.g.* formed by diffusing acceptor impurities into an n-type material.

- n-type *compensated* semiconductor: $N_d > N_a$; p-type when $N_a > N_d$.

- When $N_d = N_a$ the material is *completely compensated* and has the characteristics of an intrinsic semiconductor.

n_d is the concentration of electrons in the donor state, p_a is the concentration of holes ("empty bonding positions") in the acceptor states



Thermal equilibrium electron and hole concentrations

- In thermal equilibrium, the semiconductor crystal is electrically neutral. So for the compensated material,

$$n_0 + N_a^- = p_0 + N_d^+$$

$$\rightarrow n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

- Assume complete ionisation, n_d, $p_a = 0$, and express p_0 as n_i^2/n_0 (slide 108):

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d \rightarrow n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \quad (1)$$

(take positive root since n_0 must be positive)

Valid for n-type semiconductor or when $N_d > N_a$.

- Similarly for p-type semiconductors, or when $N_a > N_d$:

$$p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

use these equations to calculate majority carrier concentration

Thermal equilibrium electron and hole concentrations

- ***These equations are valid for extrinsic materials only – i.e. room temperature at which we assume complete ionisation***

Example: Determine thermal equilibrium electron and hole concentrations for an n-type silicon semiconductor at room temperature in which $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. Assume $n_i = 1.5\times10^{10}$ cm^{-3}

- *Answer:*

- We can see from above that if $N_d >> n_i$ then n_0 essentially equals N_d and the thermal equilibrium majority and minority carrier concentrations differ by many orders of magnitude
- **The concentration of electrons in the conduction band (majority carrier) increases above intrinsic value, n_i, as the donor impurity concentration is increased. At the same time, the minority carrier (hole) concentration decreases below n_i – WHY? …**