

Ionization Energy

- Kinetic Energy of electron is

$$T = \frac{1}{2} m_e^* v^2$$

- Substituting in velocity from equation 1 (slide 9.6) and then for r_n .

$$m_e^* v = n\hbar$$

$$r_n = \frac{n^2 \hbar^2 4\pi\kappa}{m_e^* e^2}$$

- Potential energy is (Coulomb attraction)

$$T = \frac{m_e^* e^4}{2(n\hbar)^2 (4\pi\kappa)^2}$$

$$V = \int_{R=r}^{\infty} \frac{e^2}{4\pi\kappa R^2} dR = \frac{-e^2}{4\pi\kappa r_n}$$

$$= \frac{-m_e^* e^4}{(n\hbar)^2 (4\pi\kappa)^2}$$

- Total energy ($T + V$) is therefore

$$\mathcal{E} = \frac{-m_e^* e^4}{2(n\hbar)^2 (4\pi\kappa)^2}$$

Ionization Energy

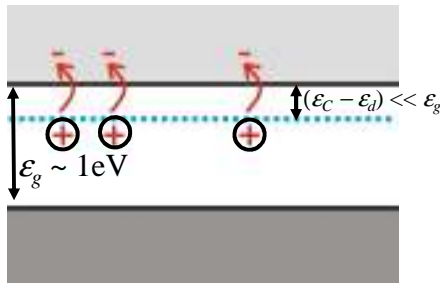
- For the hydrogen atom, $m_e^* = m_e = 9.11 \times 10^{-31}$ and $\kappa = \kappa_0 = 8.85 \times 10^{-12} \rightarrow$ ionisation energy $\mathcal{E}_1 \sim -13.6$ eV for $n = 1$.

- Compare this to silicon: **approx. ionisation energy of donor atom,**

$$\mathcal{E}_1 = (\mathcal{E}_c - \mathcal{E}_d) = -13.6 \frac{m_e^*}{m_e \kappa_r^2} \mathcal{E}_1^H \approx -0.026 \text{ eV, which is much less than the band gap of silicon } (\sim 1 \text{ eV})$$

$$\mathcal{E}_I^{Si} = \frac{m_e^*}{m_e \kappa_r^2} \mathcal{E}_I^H$$

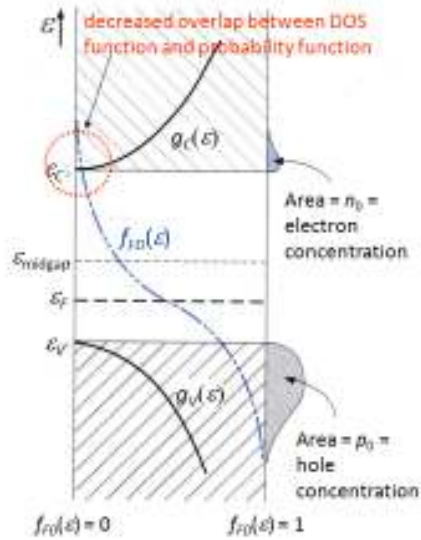
- This hydrogenic model works quite well: provides a good estimation of the magnitudes of the ionisation energies involved...



Experimentally measured
Impurity ionisation energies

	Semiconductor	
Impurity	Si	Ge
<i>Donors</i>		
Phosphorus	0.045eV	0.012eV
Arsenic	0.050eV	0.013eV
<i>Acceptors</i>		
Boron	0.045eV	0.010eV
Aluminium	0.060eV	0.010eV

Equilibrium distribution of electrons and holes: Extrinsic Semiconductor



- p-type shown opposite ($p_0 > n_0$) i.e. acceptor impurities have been added.
- Holes are the majority carrier, electrons are the minority carrier.
- The previously derived equations for n_0 and p_0 (slides 8.6 and 8.7) are still general equations in terms of Fermi energy – they are still valid.

$$n_0 = N_C \exp\left[\frac{-(\epsilon_C - \epsilon_F)}{k_B T}\right]$$

$$p_0 = N_V \exp\left[\frac{-(\epsilon_F - \epsilon_V)}{k_B T}\right]$$

Equilibrium distribution of electrons and holes: Extrinsic Semiconductor

- To derive an alternative form of the thermal equilibrium concentration equations, add and subtract an *intrinsic* Fermi energy in the exponential of the equation for n_0 (slide 8.6) i.e.
- But boxed term is *intrinsic* carrier concentration, n_i (see slide 8.8), so we write:
- Similarly for holes

$$n_0 = N_C \exp\left[\frac{-(\epsilon_C - \epsilon_{Fi}) + (\epsilon_F - \epsilon_{Fi})}{k_B T}\right]$$

$$= N_C \exp\left[\frac{-(\epsilon_C - \epsilon_{Fi})}{k_B T}\right] \exp\left[\frac{(\epsilon_F - \epsilon_{Fi})}{k_B T}\right]$$

$$n_0 = n_i \exp\left[\frac{\epsilon_F - \epsilon_{Fi}}{k_B T}\right]$$

$$p_0 = n_i \exp\left[\frac{\epsilon_{Fi} - \epsilon_F}{k_B T}\right]$$

i.e. when donor and acceptors added, ϵ_F , n_0 and p_0 change from their intrinsic values (ϵ_{Fi} , n_i and p_i)

If $\epsilon_F > \epsilon_{Fi}$, then $n_0 > n_i$ and $p_0 < n_i$; n-type with $n_0 > p_0$

If $\epsilon_F < \epsilon_{Fi}$, then $p_0 > n_i$ and $n_0 < n_i$; p-type with $p_0 > n_0$

Extrinsic Semiconductor: $n_0 p_0$ product

- Take the product of the general n_0 and p_0 expressions (slides 8.6,8.7):

$$n_0 p_0 = N_C N_V \exp\left[\frac{-(\epsilon_C - \epsilon_F)}{k_B T}\right] \exp\left[\frac{-(\epsilon_F - \epsilon_V)}{k_B T}\right]$$

$$n_0 = N_C \exp\left[\frac{-(\epsilon_C - \epsilon_F)}{k_B T}\right]$$

$$p_0 = N_V \exp\left[\frac{-(\epsilon_F - \epsilon_V)}{k_B T}\right]$$

$$n_0 p_0 = N_C N_V \exp\left[\frac{-\epsilon_g}{k_B T}\right]$$

- This equation is exactly the same as that derived for an intrinsic material on slide 8.9, therefore we have

$$n_0 p_0 = n_i^2$$

Law of mass action

- This equation states that the product $n_0 p_0$ is always constant for a given material at given temperature – it's a fundamental principle of semiconductors in thermal equilibrium.
- Note: only valid when the Boltzmann approx. valid



Degenerate and Nondegenerate semiconductors

- So far we've assumed that the concentration of dopant atoms added is small compared to density of host atoms – e.g. there is no interaction between donor electrons in *n-type* material.
- We've assumed that the impurities introduce discrete energy states. These are referred to as **nondegenerate** semiconductors.
- If impurity concentration increases, then the donor electrons (for example) will eventually interact.
- The single discrete donor energy ϵ_d will split into a band of energies, which may overlap with bottom of conduction band (when donor concentration, $N_d \sim$ effective density of states, N_C).
- When $N_d > N_C$, then ϵ_F lies within conduction band.
- **degenerate** n-type semiconductor.



Degenerate and Nondegenerate semiconductors

- Similarly for acceptor doping leading to a degenerate p-type material.
- Energy states below ϵ_F are mostly filled i.e. in the n(p)-type material most of the states between ϵ_C (ϵ_V) and ϵ_F are occupied (empty)→

electron (hole) concentration in conduction (valence) band is very large.

