TASK 2.4

$$v_F = \sqrt{2\varepsilon_F / m}$$
$$= 1.3\times10^6\,\text{ms}^{-1}$$

(thermal velocities ~ 10^5 ms^{-1})

TASK 2.5

$f_{FD}(\varepsilon)$, gives probability that an electron state will be occupied

$$f_{FD}(\varepsilon) = \frac{1}{\exp\left(\frac{3k_BT}{k_BT}\right)+1} = \frac{1}{1+20.09} = 0.0474$$

$1-f_{FD}(\varepsilon)$ gives the probability that a state will be empty

$$1-f_{FD}(\varepsilon) = 1-\frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_BT}\right)}$$

$$0.01 = 1-\frac{1}{1+\exp\left(\frac{5.95-6.25}{k_BT}\right)}$$

$$k_BT = 0.06529\text{eV}$$
$$T = 756\text{K}$$

TASK 2.6

All we have to do here is to apply the same strategy for calculating the density of states as in three dimensions.We assume periodic boundary conditions. The solid should be a square with dimensions Lx L.

$$\frac{N}{2} = \pi n_{max}^2 = \pi\left(\frac{k_{max}L}{2\pi}\right)^2$$

From this, we can write down how many electrons we can place into states within a circle of radius *k*:

$$N(k) = \frac{k^2L^2}{2\pi}$$

The density of states can be calculated as

$$g(E) = \frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE}.$$

To use this, we write

$$k = \sqrt{\frac{2m_e}{\hbar^2}}\sqrt{E},$$

$$\frac{dk}{dE} = \sqrt{\frac{2m_e}{\hbar^2}}\frac{1}{2\sqrt{E}}.$$

Inserting this gives the result

$$g(E) = \frac{L^2 m_e}{\pi\hbar^2},$$

Which is independent of energy.