

Filters

Filters are circuits or devices to remove unwanted parts of a signal. Linear filters are often described in terms of their frequency response, *i.e.* how they modify the amplitude and phase of each frequency component of the signal. Linear filters are conventionally described by their **transfer function** $\mathbf{G}(\omega)$ which is a complex function of (angular) frequency expressed as a ratio of two polynomials with complex coefficients:

$$\mathbf{G}(\omega) = \frac{\mathbf{a}_0 + \mathbf{a}_1\omega + \mathbf{a}_2\omega^2 + \dots}{\mathbf{b}_0 + \mathbf{b}_1\omega + \mathbf{b}_2\omega^2 + \dots} \quad (5.1)$$

and is the ratio of the output signal over the input signal. Much cryptic engineering-jargon arises from this convention. For example a ‘single-pole op-amp’ is one with an open-loop gain accurately described by

$$\mathbf{A}(\omega) = \frac{a_0}{\omega_0 + j\omega} \quad (5.2)$$

which has a single complex pole at $\omega = j\omega_0$.

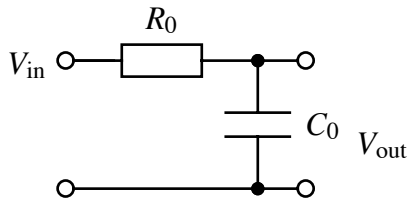
Passive Filters

Passive filters are constructed entirely from R , L and C elements and therefore require no power supply. The most common is the low-pass first-order RC-filter, circuit 5.1, which attenuates at a rate of 6 dB per octave above its corner at $f_t = 1/2\pi R_a C_a$. Circuit 5.2 uses an inductor to achieve a rapid transition between the **pass-band** (*i.e.* the region of constant amplitude gain) below f_t and the **stop-band** above it where the attenuation is 18 dB per octave. There is no single criterion for ‘best’ so there is an enormous number of types of filter; the most commonly encountered include: Butterworth (maximum flatness in pass-band), Chebyshev (sharpest transition from pass-band to stop-band), Bessel (linear phase response).

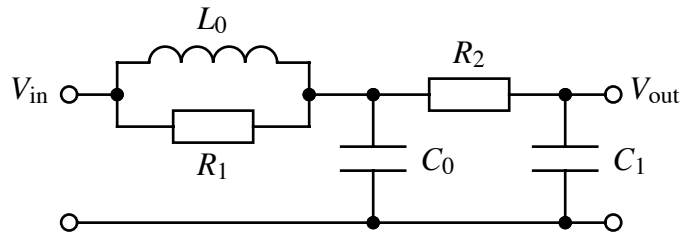
Required Reading

Amplification – Storey (1998) §3.6–3.7 pp. 66–83 / (2006) §3.6–3.9 pp. 61–82.

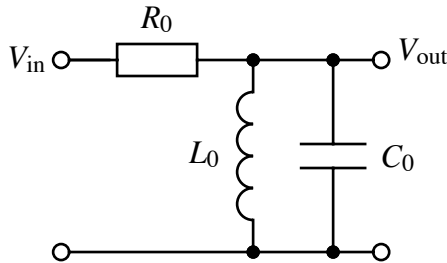
Positive Feedback – Storey (1998) §8.1–8.2 pp. 312–323 / (2006) §11.1–11.2 pp. 350–356.



Circuit 5.1



Circuit 5.2



Circuit 5.3

R_0	10K
R_1	14K
R_2	100K
C_0	15n9
C_1	1n59
L_0	1H59

Exercise 5.1 What is the magnitude of the signal represented by

$$\mathbf{V}_{out}(t) = \frac{(a + jb)}{c + jd} \exp(j\omega t)$$

and what is its phase relative to the signal $\mathbf{V}_{in}(t) = b \exp(j\omega t)$?

Answer $|\mathbf{V}_{out}| = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}} \quad |\phi| = \arctan\left(\frac{bc - ad}{ac + bd}\right)$

Exercise 5.2 Calculate an expression, in the form of equation 5.1, for the transfer functions of (a) circuit 5.1, and (b) circuit 5.2. In the first case also obtain the magnitude and phase. In the second case, give up after ten minutes and think ‘there must be an easier way...’

Answer (a) $\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1}{1 + j\omega C_0 R_0} \quad \left| \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} \right| = \frac{1}{(1 + \omega^2 C_0^2 R_0^2)^{1/2}} \quad \phi = \arctan(-\omega C_0 R_0)$

(b) $\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{\alpha + \beta j\omega}{\alpha + B j\omega - C\omega^2 - D j\omega^3} \quad \alpha = R_1 R_2 \quad \beta = L_0 R_2 \quad B = C_1 R_1 R_2^2 + L_0 R_2$
 $C = L_0 R_2 [C_1 (R_1 + R_2) + C_0 R_1] \quad D = C_0 C_1 L_0 R_1 R_2^2$

Exercise 5.3 How is the bandwidth of a filter defined? Obtain the transfer function of Circuit 5.3 and hence calculate its bandwidth

Answer -3 dB points at 618Hz and 1616Hz so bandwidth is about 1 kHz.