Nonlinear Circuits and Devices

Linearity

The response $\mathbf{F}(\mathbf{X})$ of system to an applied signal $\mathbf{X}$ is said to be **linear** if

$$\mathbf{F}(\mathbf{X}_1 + \mathbf{X}_2) = \mathbf{F}(\mathbf{X}_1) + \mathbf{F}(\mathbf{X}_2) \tag{7.1}$$

For example, the current flowing through a resistor that obeys Ohm's law is a linear function of the voltage across it because, in this case, its resistance R is a constant. The current flowing in every branch, and the voltage at any node, of a network comprising solely linear impedances can be found exactly by solving a set of linear equations. However, if the network contains one or more nonlinear elements, there is no guarantee that it can be solved, even in principle (*e.g.* it may exhibit chaotic behaviour).

Solving Nonlinear Circuits by Brute Force

The availabilty of powerful computers has made it possible to solve complicated nonlinear circuits by brute force. The procedure starts with the circuit in a known condition (*e.g.* all voltages and

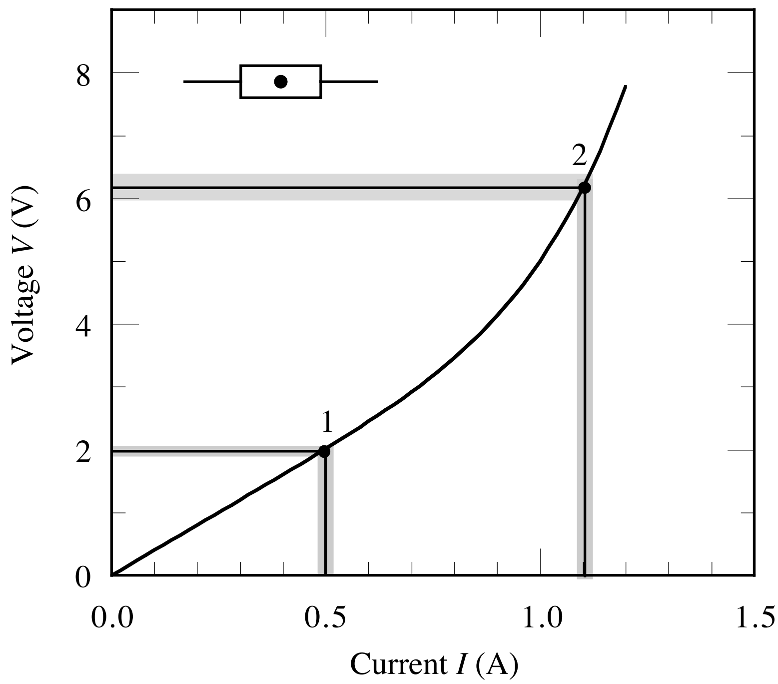


Figure 7.1 V-I curve for a nonlinear resistance

inputs zero) at time $t = 0$ and then takes small steps forward in time. At each time-step the voltage at every node is found by integrating the differential equations describing the time-domain response of the components that connect the nodes, subject to the constraint of charge conservation. The accuracy of this method, which is called **transient analysis**, depends critically on the fidelity of the device models, the sophistication of the differential equation solver used, and the analyst having the patience to chose a sufficiently small time-step.

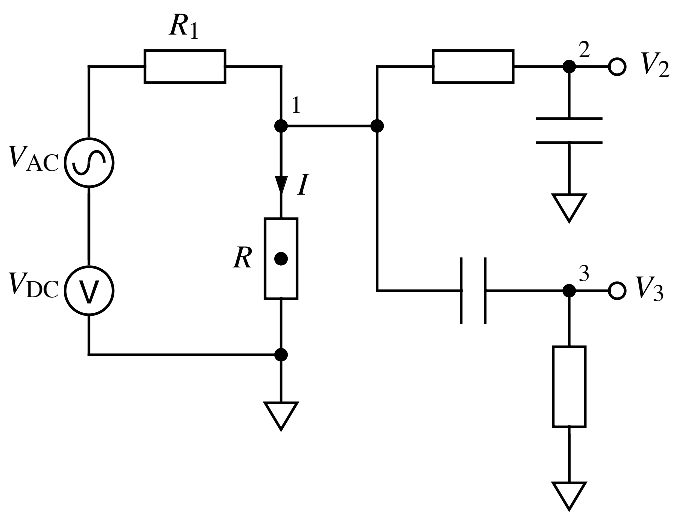
Solving Nonlinear Circuits by Perturbation Theory

The traditional method of analysing nonlinear circuits starts by using an iterative procedure to find the **DC operating-point** of each node, *i.e.* its steady-state voltage when none of the inputs to the system are changing with time. Taylor's Theorem is then invoked to show that, provided they are of small enough amplitude, any AC signals superimposed on the input terminals will result in a linear response. Once calculated, this small-signal analysis can be used to derive the system transfer function, noise and, by using Fourier Transforms, the time-domain response.

Circuit 7.1 incorporates the nonlinear resistor characterised in figure 7.1. It will be assumed that the output filters require a negligible current to flow from node 1 to nodes 2 and 3. The Taylor expansion of the voltage V across the nonlinear device R,

$$V(I_0 + i) \approx V(I_0) + i\left(\frac{dV}{dI}\right)_{I_0} = I_0 R(I_0) + ir \tag{7.2}$$

is used to define the **differential resistance** r of the device R. In this case, the small time-varying current i is



Circuit 7.1 Investigating the nonlinear element R

$$i = I(t) - I_0 = \frac{(V_{AC}(t) + V_{DC})}{R(t) + R_1} - \frac{V_{DC}}{R(I_0) + R_1} \approx \frac{V_{AC}(t)}{R(I_0) + R_1}. \tag{7.3}$$

The DC signal measured at node 2, and the AC signal at node 3, will therefore be

$$V_2 = V_{DC} \frac{R(I_0)}{R(I_0) + R_1} \quad \text{and} \quad \mathbf{V}_3 = r(I_0) \frac{\mathbf{V}_{AC}}{R(I_0) + R_1}. \tag{7.4}$$

Nonlinear Amplifiers

Circuits 7.2 and 7.3 comprise an ideal op-amp, and a nonlinear device X with a voltage-current relationship defined by $V_X(I) = f(I)$. When voltage V_{in} is applied to circuit 7.2 the output V_{out} must satisfy

$$V_{out} = -f(V_{in}/R) \tag{7.5}$$

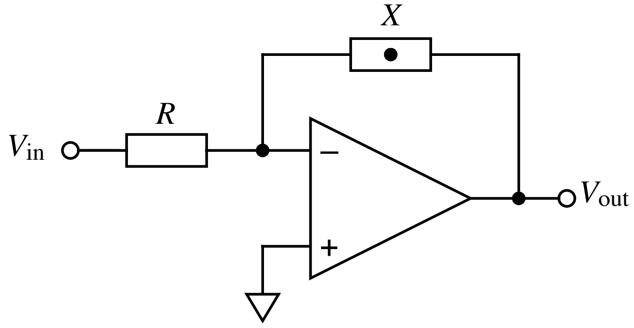
and in the case of circuit 7.3

$$f(V_{out}/R) = -V_{in} \quad \Rightarrow \quad V_{out} = -Rf^{-1}(V_{in}) \tag{7.6}$$

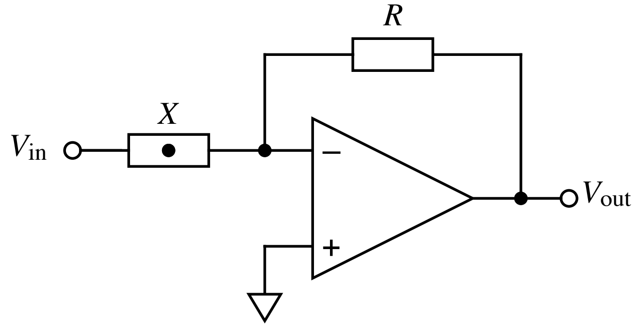
Such circuits can be difficult to get working well as the amplitude-dependent gain can cause problematic stability.

Required Reading

Semiconductors and Diodes – Storey (1998) §5.1–5.9 pp. 156–180 / (2006) §6.1–6.8 pp. 141–167.



Circuit 7.2 Nonlinear amplifier



Circuit 7.3 Inverse nonlinear amplifier

Exercise 7.1 A nonlinear device has a voltage-current relationship $V = \alpha I + \beta I^2$ where $\alpha = 3\ \mathrm{k\Omega}$ and $\beta = 5\ \mathrm{k\Omega\ mA^{-1}}$. It is used as device R in circuit 7.1 with $R_1 = 7\ \mathrm{k\Omega}$. Assume the high- and low-pass filters are ideal and no current flows from node 1 to nodes 2 or 3.

(a) Find V_2 and the current I that flows when $V_{DC} = 1\ \mathrm{V}$ and $V_{AC} = 0\ \mathrm{V}$.

(b) Calculate the differential resistance of the device when $V_{DC} = 1\ \mathrm{V}$ and hence find the amplitude of V_3 if $V_{AC} = 1\ \mathrm{mV}$

(c) If $V_{DC} = 0\ \mathrm{V}$ and V_{AC} is a sinewave of amplitude 0.1 V and frequency 1 kHz, calculate expressions for the DC voltage at node 2 and the AC signal at node 3.

Answer (a) $I_0 = 95\ \mu\mathrm{A}$ $\therefore$ $\mathrm{V}_2 = V_1 = 0.33\ \mathrm{V}$ (quadratic eqn for I and take +ve root)

(b) $V_3 = 377\ \mu\mathrm{V}$

(c) $V_2 = 1.7\ \mathrm{mV}$ $V_3 = 30[\mathrm{mV}]\sin(\omega t) - 1.7[\mathrm{mV}]\cos(2\omega t)$

Exercise 7.2 A diode has a current-voltage characteristic of $I \approx I_0 \exp(eV/1.5k_BT)$. Explain how you could use it as the basis of a circuit which had $V_{out} = -[1\ \mathrm{V}]\log_{10}(V_{in}/[1\ \mathrm{V}])$.

Answer A 3 op-amp design in which two simple log-amplifers are subtracted.