Drill Problems

Introduction

Drill problems are short exercises to help you to practise the techniques discussed in the lectures. To derive real benefit from these questions you should try to do them as soon as possible after the lecture that covers the material. If you have difficulty doing these problems it might indicate that you are not understanding what is going on in the lectures; first read the textbook and check the hints on the separate handout and if you are still stuck seek help from your study group or tutor without delay. Problems marked with an asterisk "*" are more interesting, but much harder than the usual standard; there is no need to worry if you can't do them!

Notation

Vectors are conventionally indicated by boldface type in printed material, *e.g.* **B**. In hand-written materials under- or over-lining is used, *e.g.* $\underline{B}$, $\overline{B}$, or $\vec{B}$. The unit vector in the direction **B** is printed $\hat{\mathbf{B}}$ and hand-written $\hat{\underline{B}}$ or $\hat{B}$. Some authors use $\mathbf{i}, \mathbf{j}, \mathbf{k}$ for the unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ in the Cartesian coordinate system; avoid this notation as **j** is more often used for the current density field.

DP1 In rectangular Cartesian coordinates (x,y,z) a scalar field ϕ has the form

$$\phi = xyz^2 - x^2yz\,.$$

Calculate $\nabla\phi$ at the point P having coordinates (1,2,3).

DP2 The vector $\mathbf{r}$ is defined in spherical polar coordinates by $\mathbf{r} = r\hat{\mathbf{r}}$ and in Cartesian coordinates by $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$.

(a) Express the function $\phi = A/|\mathbf{r}|$, where A is constant, in terms of (a) r and (b) x,y,z.

(b) Use the definitions of ∇ given on the "Vector Analysis Formulae" handout to find $\nabla\phi$ in both coordinate systems. Are the expressions equal, and if so, why?

DP3 A frame of reference S' is rotated by 45° clockwise about the $\hat{\mathbf{z}}$-axis of another frame S. This means that the frames are related by the transformations

$$\hat{\mathbf{x}}' = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}}); \quad \hat{\mathbf{y}}' = \frac{1}{\sqrt{2}}(-\hat{\mathbf{x}} + \hat{\mathbf{y}}); \quad \hat{\mathbf{z}}' = \hat{\mathbf{z}}.$$

Find the 3×3 matrix that transforms the components in S of a vector $\mathbf{C}$ into its components in S'. If the components of $\mathbf{C}$ in S' are (0.7,2,1.5) what are its components in S?

DP4 What is the divergence of the vector fields when a and b are constants:

(a) $\mathbf{G} = ax^2\hat{\mathbf{x}} + by\hat{\mathbf{y}} + \hat{\mathbf{z}}$ in Cartesian coordinates?

(b) $\mathbf{H} = ar^2\hat{\mathbf{r}} + br\theta\hat{\boldsymbol{\theta}} + \theta\hat{\boldsymbol{\varphi}}$ in spherical polar coordinates?

DP5 Charge is moving in a system causing a current density $\mathbf{j} = ax^2\hat{\mathbf{x}} + by\hat{\mathbf{y}}$. What is happening to the charge density at the point $(2\hat{\mathbf{x}} + 3\hat{\mathbf{y}})\,\mathrm{m}$ if the parameters have the values:

$$a = 12\,\mathrm{Am}^{-4}; \quad b = 4\,\mathrm{Am}^{-3}.$$

DP6 What is the net electric flux through the surface of an empty $1\,\mathrm{m}$ cube, centred on the origin, in an electric field $(90\hat{\mathbf{x}} + 75\hat{\mathbf{z}})\,\mathrm{Vm}^{-1}$?

DP7 A cube of side length L, centred on the origin of the coordinates, is bounded by the closed surface S. Find the total flux of the vector field $\mathbf{F} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ out of the surface S by:

(a) Evaluating the flux through each face of the cube separately.

(b) Using the divergence theorem.

DP8 A circle of radius R in the $z=0$ plane is defined by $x^2 + y^2 = R^2$.

(a) Obtain an expression for the vector line element d**l** of the circle in terms of dx the associated differential change in x.

(b) Evaluate the closed line integral $\oint \mathbf{F} \cdot \mathrm{d}\mathbf{l}$ around the circle, for the case

$$\mathbf{F} = \frac{y\hat{\mathbf{x}} - x\hat{\mathbf{y}}}{\left(x^2 + y^2\right)}.$$

DP9 The velocity field of a fluid in cylindrical polar coordinates is $\mathbf{v}(r,\theta,z) = \omega r\hat{\boldsymbol{\theta}}$ where ω is a constant. What are the units of ω and how could such a flow be established in water? Calculate (a) the curl of the velocity field, and (b) the *circulation* around a circular path of radius r around the z-axis. How are the two quantities related?

DP10 Use the Cartesian form of ∇ to find expressions for $\mathrm{Curl}\,\mathbf{F} = \nabla \times \mathbf{F}$ for the fields

$$\text{(a) } \mathbf{F} = \frac{y\hat{\mathbf{x}} - x\hat{\mathbf{y}}}{\left(x^2 + y^2\right)^{3/2}} \qquad \text{and} \qquad \text{(b) } \mathbf{F} = \frac{y\hat{\mathbf{x}} - x\hat{\mathbf{y}}}{x^2 + y^2}.$$

DP11 Use the Cartesian form of ∇ to show that, when **A** is a vector field and f is a scalar field:

(a) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$, (b) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, (c) $\nabla \times (\nabla f) = 0$.

DP12 The potential $\phi(\mathbf{r})$ at a position **r** due to a point charge q at position $\mathbf{r}'$ is

$$\phi(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\mathbf{r} - \mathbf{r}'|}.$$

Calculate $\nabla\phi$ and hence find the electric field **E** at any position **r**.

DP13 Find an approximation for $V(\mathbf{r})$ that is valid when $d \ll r$ and

$$V(\mathbf{r}) = \frac{1}{|\mathbf{r} - \mathbf{d}|}.$$

DP15 (a) Find the monopole and dipole moments of this arrangement of charges.

(b) Does it have a non-zero quadrupole moment?

(c) At what distance has the maximum dipole contribution to its **E** field dropped to 10% of the monopole field?

DP16 What are the SI units of **D**, **E**, **P** and ε_0?

DP17 If $\delta(x)$ is the Dirac delta function evaluate

(a) $\int_{-\infty}^{+\infty} \sin(x)\,\delta(x-0.34)\,\mathrm{d}x$ (b) $\int_{-\infty}^{+\infty} \sin(x)\left[\frac{\mathrm{d}}{\mathrm{d}x}\delta(x-0.34)\right]\mathrm{d}x$.

DP18 Two charges, $+q$ and $-q$ are positioned at $+\mathbf{d}/2$ and $-\mathbf{d}/2$ respectively. Calculate the change in the potential energy of this system when a uniform field **E** is applied and hence confirm the expression stated in the lectures for the potential energy of a point dipole **p** in a uniform electrostatic field.

DP19 Two pieces of dielectric material are bonded together with the interface lying in the plane $x=0$. Find the equivalent bound surface charge density at the interface if the polarisations in the materials are:

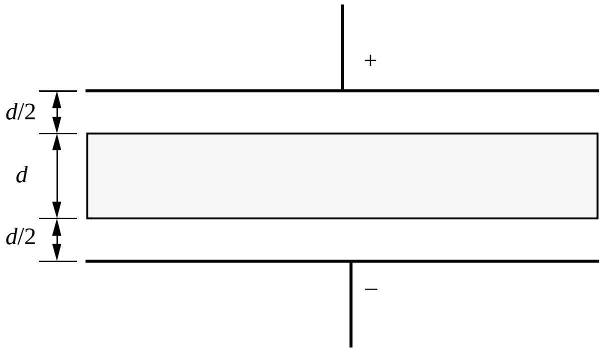
$$\mathbf{P}(x<0) = 10\hat{\mathbf{x}}\ \mathrm{Cm}^{-2} \quad \text{and} \quad \mathbf{P}(x>0) = 3\hat{\mathbf{x}}\ \mathrm{Cm}^{-2}.$$

DP20 A sphere of dielectric material is uniformly polarised in the z-direction, *i.e.* $\mathbf{P} = \hat{\mathbf{z}}P$. Find the *equivalent bound surface charge density* $\sigma(\theta,\varphi)$ at the point on its surface specified by spherical polar coordinates (θ,φ).

DP21 Find the field at the centre of the sphere described in the previous problem by using a surface integral to calculate the field due to the charge distribution $\sigma(\theta,\varphi)$.

DP22 If liquid helium has a density $\rho = 145\,\mathrm{kgm^{-3}}$ and relative permittivity $\varepsilon_r = 1.0556$ what is the the atomic polarisability of helium?

DP23 A slab of dielectric, relative permittivity ε_r, is placed between the plates of a capacitor as shown. A potential difference V is maintained between the plates. Ignore edge effects and calculate the values of **D**, **E** and **P** (a) inside the dielectric (b) in the gaps between the dielectric and the plates.



DP24 Calculate the charge density on the plates of the capacitor described in the previous problem. Multiply this by the area of each plate A to find the total charge and hence deduce the capacitance of the structure.

DP25 Deduce Poisson's equation for electrostatics from the differential equations relating the electric-field intensity **E** to the charge density ρ and the potential ϕ.

DP26 A long coaxial cable consists of an inner wire, radius r, inside a metal tube of internal radius R, the space in between being completely filled with a dielectric of relative permittivity ε_r. Use Gauss's law to find the electric displacement flux **D** and hence show that the cable capacitance is $\left(2\pi\varepsilon_r\varepsilon_0\right)/\ln(R/r)$.

DP27 In free space $\mathbf{D} = \varepsilon_0\mathbf{E}$ so the expression for the energy associated with a field **E** in a region V simplifies to

$$U = \tfrac{1}{2}\varepsilon_0\int_V \mathrm{E}^2\,\mathrm{d}^3 r \quad .$$

Use this expression to find the electrostatic energy stored in terms of the plate area A, separation d and the internal electric field strength E in a parallel-plate air capacitor. Hence find the energy stored in a capacitor in terms of the potential difference and its capacitance.

DP28 Six identical spherical drops of mercury are each charged to 10 V above earth potential and then made to coalesce into a single spherical drop. If the drops were initially widely separated, what is the potential of the new drop? Has the electrostatic energy changed?

DP29 Use Laplace's equation in spherical polar coordinates to find the potential as a function of radius in the space between two concentric conducting sphere of radius a and b and at potentials V_1 and V_2 respectively.

DP30* Find an expression for the potential at a perpendicular distance a from the centre of a straight line of total charge Q spread along a length l. Hence show that the computer program that calculated the values in table 3-3 of Reitz Milford and Christy must have had a "bug" in it.

DP31* Cork is a dielectric with relative permittivity 3.6 and density of $2.5\times10^{2}\,\mathrm{kgm^{-3}}$. A small sphere of it is suspended a distance d vertically below a point electric charge of $10^{-7}\,\mathrm{C}$. Estimate the value of d at which the cork sphere will be picked up by the point charge.

DP32 A 15 pF parallel-plate air capacitor is charged by connecting it to a 75 V PSU. How much work must be done to double the separation of the plates of the capacitor (a) with the PSU disconnected and the capacitor fully charged (so the charge stays constant), and (b) with the PSU connected (so the potential difference stays constant)?

DP33* A parallel plate capacitor has rectangular plates of length a and width b spaced d apart connected to a battery of EMF V. A slab of dielectric of relative permittivity ε_r that would just fill the space between the capacitor plates is slid part way between them. Show that the force pulling it into the plates is $\varepsilon_0\left(\varepsilon_\mathrm{r}-1\right)V^2b/2d$. Why is the answer independent of a?

DP34 A conducting sphere of radius R is placed in a uniform electric field $\mathbf{E}_0$. Check that the potential ϕ at a point (r,θ) outside the sphere is given by

$$\phi = -E_0 r\cos\theta\left\{1-(R/r)^3\right\}$$

as follows: (a) Does the potential satisfy Laplace's equation? (b) Is the potential the same at all points on the surface of the sphere? (c) Is the electric field far from the sphere constant?

DP35 In cylindrical polar coordinates, the electric charge on each element of surface area of an earthed infinite conducting plane induced by a point charge q a distance z above it is $dQ = \sigma r \mathrm{d}r \mathrm{d}\theta$, where the induced surface charge density σ is

$$\sigma = \varepsilon_0 \mathbf{E} \cdot \hat{\mathbf{z}} = \frac{-qz}{2\pi\left(z^2 + r^2\right)^{3/2}}.$$

Apply Coulomb's law to q and dQ and show by integration that the total force acting on the point charge q is that which would be exerted on it by its image charge in the absence of the conducting plane.

DP36 Use the Biot-Savart formula to calculate the magnitude and direction of the magnetic field at the centre of a short solenoid of radius 30 mm and length 25 mm carrying a current of 7A and having 500 turns.

DP37 The current density in a long straight conductor of circular cross-section and diameter d varies with radius as $\mathbf{j} = j_0 r^2 \hat{\mathbf{z}}$. Calculate the current flowing in the conductor.

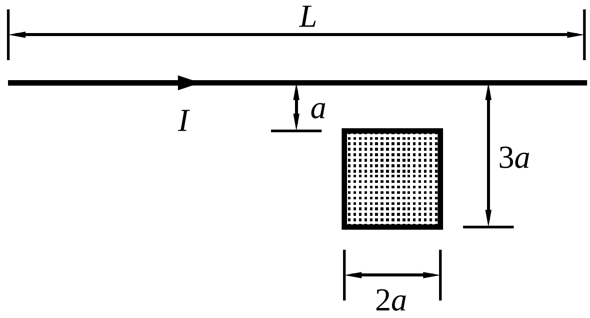
DP38 Use symmetry and the integral form of Ampère's Law to find an expression for the field $\mathbf{B}$ (for all $r > 0$) due to the conductor described in the previous question. The expression should be in cylindrical polar coordinates. Check that your answer is correct by using the differential form of Ampère's Law.

DP39 A long thin wire, length L, in free space carries a current I. Provided $L >> r$ the magnitude of the vector potential $\mathbf{A}$ at a distance r from the midpoint of the wire on a line perpendicular to its axis is

$$|\mathbf{A}| = \frac{\mu_0 I}{2\pi} \ln\left[\frac{L}{r}\right].$$

What is the direction of the vector $\mathbf{A}$?

DP40 How is the magnetic flux through a loop related to the vector potential? Use the result of the previous question to find the flux through the square loop shown in the diagram.



DP41 A current of 1 A flows in circular loop with its axis along the $\hat{\mathbf{z}}$ direction and diameter 10 mm. Find the magnetic dipole moment **m** of the loop. What are the maximum and minimum values of the magnetic field **B** due to this loop at a distance 100 mm from its centre?

DP42 A single-turn coil of wire has an area of 1 cm^2 and carries a constant current of 1 A. If it is placed in a very uniform field of 1 T, (*e.g.* in an MRI magnet) what net force does it experience due to the field? How does the torque required to rotate it slowly in the field vary with angle of rotation?

DP43 A permanent magnet has the shape of a right cylinder of radius a and length L. The magnetisation **M** is uniform and along the direction of the cylinder axis. (a) Find the volume and surface equivalent current densities for the magnetisation, and (b) compare the current distribution with that of a solenoid.

DP44 In a magnetic field when $\mathbf{H} = 5\times10^4\,\mathrm{Am^{-1}}$, iron is magnetically saturated (*i.e.* $\mathbf{M} = \mathbf{M}_S$, its maximum value for the medium) and the magnetic **B** field is 2.20 T. Calculate: (a) the magnetisation of iron; (b) The **B** field when $H = 1\times10^5\,\mathrm{Am^{-1}}$.

DP45 The saturation magnetisation $\mathbf{M}_S$ for iron is $1.7\times10^6\,\mathrm{JT^{-1}m^{-3}}$ which can be thought of as arising when the dipole moment of every atom points in the same direction. Find the effective permanent magnetic dipole moment of an iron atom (in Bohr magnetons) assuming that iron has a relative atomic mass of 56 and a density of $7.9\times10^3\,\mathrm{kgm^{-3}}$.

DP46 An electromagnet consists of a soft iron ring of mean radius 75 mm with an air gap of 3 mm. It is wound with 50 turns carrying a current of 6 A. Given that the B-H curve for soft iron has the following characteristics, determine the magnetic field in the gap:

B (T)	0.2	0.4	0.6	0.8	1.0
H (Am^{-1})	100	170	220	310	500

DP47 An aircraft flies at a constant speed of 330 ms^{-1} at an angle of 60° to Earth's magnetic which has magnitude 0.5×10^{-5}T. (i) What magnitude of electric field will be established inside the electrically conducting material of the aircraft to exactly cancel the magnetic force? (ii) What is (a) the potential difference, and (b) the EMF, across the wing tips?

DP48 A current loop of area S in the xy-plane is placed in a uniform magnetic field $B_z = B_0 \sin \omega t$. Find the EMF induced in the coil when: (a) it is fixed; (b) it rotates at an angular frequency ω about the x-axis.

DP49 A long cylindrical solenoid, radius $r = 100$ mm, produces a magnetic **B** field along the solenoid axis that can be taken to be uniform inside the solenoid and zero outside. What expression can be used to find $\nabla \times \mathbf{E}$ when **B** changes with time? Find the magnitude of the transient electric field produced at the surface of the solenoid when the **B** field is ramped at a linear rate from 0 T to 8 T over a period of 5 s. Sketch the electric field.

DP50 If the loop described in DP39 has a resistance $R = 20\Omega$ and the current I flowing in the straight wire increases at a rate of 1 As^{-1} what current is induced in the loop if its area is 1 cm^2? Ignore the self-inductance of the loop, *i.e.* the field due to induced currents circulating. Under what circumstances would you expect this to be a poor approximation?

DP51 A solenoid of 1000 turns is 25 mm in diameter and 100 mm long. Estimate the energy stored in it when it is carrying 100 A.

DP52 Calculate the total magnetic energy in Earth's external field by assuming that it is due to a dipole at the its centre generating a magnetic field of 20 μT at the equator.

Drill Problems – Hints and Answers

DP1 $\nabla\phi(1,2,3) = 6\hat{\mathbf{x}} + 6\hat{\mathbf{y}} + 10\hat{\mathbf{z}}$.

DP2 The magnitude of the vector **r** can be calculated in any coordinate system from the definition $|\mathbf{r}| = (\mathbf{r}\cdot\mathbf{r})^{1/2}$ *i.e.* $|\mathbf{r}| = r$ in spherical polar coordinates and $|\mathbf{r}| = \sqrt{x^2+y^2+z^2}$ in Cartesian coordinates.

DP3 (–0.919,1.909,1.5).

DP4 (a) $2ax+b$, (b) $4ar + b(1+\theta\cot\theta)$.

DP5 Apply the continuity equation derived in section 2.4 of the notes. The charge density is *decreasing* with time at a rate of $52\,\mathrm{Cm^{-3}s^{-1}}$.

DP6 The integral form of Gauss's Law solves this trivially – if net enclosed charge is zero the net electric flux is also zero.

DP7 Answer: $3L^3$. With this particular field it doesn't matter how the cube is orientated so choose an orientation that makes the integrals easy.

DP8 Use the substitutions $x = R\cos\theta$ and $y = R\sin\theta$ to find dx and dy in terms of dθ then (a) $d\mathbf{l} = \left(-\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{y}}\cos\theta\right)R\mathrm{d}\theta$. and (b) -2π.

DP9 (a) Use the expression in cylindrical coordinates, only one component is non zero $\nabla\times\mathbf{v} = 2\omega\hat{\mathbf{z}}$, (b) $\oint\mathbf{v}\cdot d\mathbf{l} = 2\pi r^2\omega$.

DP10 (a) $\dfrac{-3}{\left(x^2+y^2\right)^{3/2}}$ (b) $\dfrac{-2}{\left(x^2+y^2\right)}$.

DP11 Only (a) is very interesting – it is important to realise that $\nabla(\nabla\cdot\mathbf{A}) \neq \nabla^2\mathbf{A}$.

DP12 As far as the ∇ operator is concerned $\mathbf{r}'$ is a constant so applying identity VA–17 gives

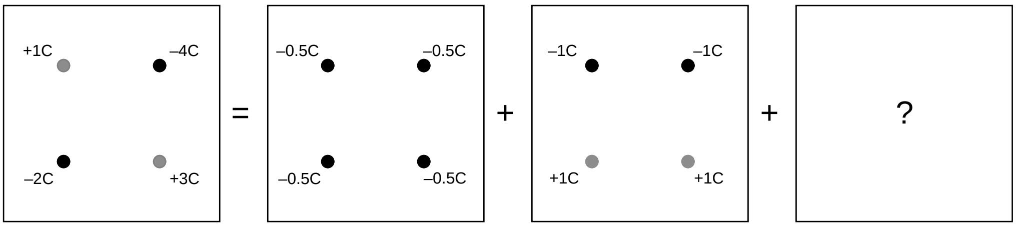
$$\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = \frac{-(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \text{ and therefore } \mathbf{E} = -\nabla\phi(\mathbf{r}) = \frac{-q}{4\pi\varepsilon_0}\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = \frac{q}{4\pi\varepsilon_0}\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}.$$

DP13 Remember that $|\mathbf{A}| \equiv A = (\mathbf{A}\cdot\mathbf{A})^{1/2}$ and use the binomial expansion. The details are worked out on the "Multipole Expansions" handout with $\mathbf{r}'$ instead of **d**.

DP14 The monopole moment is just the net charge $\int\rho(\mathbf{r})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$. The delta functions integrate to unity in the *x* and *y* dimensions leaving a simple integral along the *z* axis. The dipole moment about the origin is found by evaluating

$$\int \mathbf{r}\rho(\mathbf{r})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \int \left(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}\right)\rho(\mathbf{r})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,.$$

DP15 (a) $-2\,\mathrm{C}$, $-2\times 10^{-9}\,\hat{\mathbf{y}}\ \mathrm{C\ m}$. (b) You could just calculate the elements of the quadrupole tensor but there is a less tedious method using the superposition principle to subtract off the known monopole and dipole contributions and see what's left.



DP16 Use formulae you know are correct such as $\mathbf{F} = q\mathbf{E}$ for the force on a charge, Coulomb's law and $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$.

DP17 (a) 0.333, (b) 0.943 .

DP18 Write expressions for the work done moving the charges one-by-one from infinity to the origin and then to their final positions. The work done against **E** to get to the origin is of opposite sign in each case and so sums to zero and the Coloumb interaction between the charges is a constant that doesn't depend on **E**.

DP19 Look at the derivation on EM-29 and EM-30 which deals with the interface between a substance and the vacuum where $\mathbf{P}=0$. How must it be modified to handle the interface between two regions of differing **P**?

DP20 This is a sphere of dielectric material in a vacuum so the direction of the vector normal to the surface is a function of θ and φ. The example given in the lecture dealt with a sphere of vacuum inside a dielectric of infinite extent.

DP21 The example given in the lecture dealt with a sphere of vacuum inside a dielectric of infinite extent so the result of the integration can be checked by using the principle of superposition. Answer: $\mathbf{E} = \mathbf{E}_0 - \mathbf{P}/3\varepsilon_0$ where $\mathbf{E}_0$ is the external applied field.

DP22 The relative atomic mass of helium is 4 so start by calculating the number density N (atoms per unit volume). Then use the Claussius-Mossotti equation to relate the macroscopic dielectric constant to the atomic polarisibility.

DP23 $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ and $\mathbf{P}=0$ in the gap and integrating along a path joining the two plates gives $\int \mathbf{E}\cdot d\mathbf{l} = V$. The normal component of **D** is continuous across the interface between dielectric and air where there is no free charge. Failing this look at the back of the handout on Electrostatic boundary conditions.

DP24 Apply Gauss's law in integral form to a pill box surface partly embedded in the conducting plate. Inside the metal $\mathbf{E}=0$. Recall that C is defined by $Q=CV$.

DP25 This was done in the lectures in section 2.9

DP26 Use the integral form of $(\nabla \cdot \mathbf{D}) = \rho_{\text{free}}$ and recall that ρ_{free} is zero everywhere except on the metal surfaces. Also recall that $\mathbf{D} = \varepsilon_{\text{r}}\varepsilon_0\mathbf{E}$ and $\int \mathbf{E} \cdot d\mathbf{l} = V$.

DP27 Ignore the fringe fields and assume that $E = V/d$ between the plates and zero elsewhere. Then $U = \frac{1}{2}\varepsilon_0 Ad(V/d)^2$ and therefore $U = \frac{1}{2}CV^2$.

DP28 What is the charge q_0 on a conducting sphere of radius r_0 at potential V_0? What is the energy stored in its field? Answer: 3.03V, and the energy increases by a factor 3.3

DP29 There is spherical symmetry so

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = 0 \quad \Rightarrow \quad \left(r^2\frac{\partial\phi}{\partial r}\right) = k_1 \quad \Rightarrow \quad \phi(r) = k_2 - \frac{k_1}{r}$$

and the constants are chosen to fit the boundary conditions by solving two simultaneous equations.

DP30 Although the RMC problem involves a rod that is not uniformly charged correcting for this would make the agreement worse rather than better. I suspect that the programmer assumed that $\log(\mathrm{x}) \equiv \ln(x)$ but the computer used $\log(\mathrm{x}) \equiv \log_{10}(x)$.

DP31 Assume that the cork sphere is small and find the induced electric dipole due to the field, then find the accompanying electric force due to the field gradient.

DP32 (a) 4.2×10^{-8}J, and (b) 2.1×10^{-8}J [The work done on the PSU by the charge flowing through it needs to be included in the calculation.]

DP33 This sort of problem can usually be solved by obtaining an expression for the energy of the system as a function of the position of the moving part and then finding the force by differentiation.

DP34 Use expressions for $-\nabla\phi$ and $\nabla^2\phi$ in spherical polar coordinates. $R = r$ defines the surface of the sphere and $r \to \infty$ gives the potential far away from the sphere.

DP35 $\mathbf{F} = \dfrac{-\hat{\mathbf{z}}}{16\pi\varepsilon_0}\left(\dfrac{q}{z}\right)^2$

DP36 $B=6.8\times10^{-3}$T

DP37 $I = j_0\left(\pi d^4/32\right)$

DP38 $\mathbf{B} = \begin{cases} \left(d^4/64r\right)\mu_0 j_0\hat{\boldsymbol{\theta}} & \text{for} \quad r > d/2 \\ 0.25r^3\mu_0 j_0\hat{\boldsymbol{\theta}} & \text{for} \quad r < d/2 \end{cases}$

DP39 Along the same direction that I is flowing.

DP40 Section 5.3 of lecture notes (OHP-EM70). $\Phi = a\mu_0 I \ln(2)/\pi$

DP41 Use the definition in section 5.4 of the lecture notes (OHP-EM71).

DP42 It is the *gradient* of the field that exerts a force on the coil lecture notes section 5.4 and the torque can be found from differentiating the "pseudo potential energy" with respect to angle or using equation 23 on the *Small Current-Loops* handout.

DP43 Check the definitions in section 5.6 of the lecture notes (OHP-EM75).

DP44 Remember that the magnetisation is the "dipole moment per unit volume" and use the definition $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$.

DP45 Calculate the number of atoms per cubic metre of iron and look up the definition of the Bohr magneton in a reference book such as Kaye and Laby.

DP46 This problem is an important exercise in applying the boundary conditions for **B** and **H**, it is the magnetic equivalent of problem DP23. Diagram and help in section 9-9 of RMC.

DP47 When the aircraft starts flying the conduction electrons experience a Lorentz force and move from one wing to the other. Read RMC 11-1 carefully before answering part (ii) as a circuit and a reference frame need to be specified as part of the answer.

DP48 Find the flux through the coil in each case an apply Faraday's law.

DP49 The expression is one of Maxwell's equations; Faraday's law in point form. The electric field tries to oppose the increase in current in the wire.

DP50 The flux through the loop is related to a line integral of the vector potential around the loop. The rate of change of this with time gives the EMF.

DP51 Assume that **B** is constant everywhere in the solenoid and negligible outside.

DP52 Find the expression for the field due to a magnetic dipole, *e.g.* a small current loop. A "point on the equator" is at $r = R_{\text{Earth}}$ and $\theta = \pi/2$. Assume the radius of the earth is $6400\,\text{km}$ and integrate the dipolar field external to the earth.