Dielectric Tensors and Constants

Real dielectrics can be very complicated. A general expression for the components of the polarisation **P** at a point where the field is **E** is

$$P_i = P_{0i} + \varepsilon_0 \sum_j \chi_{ij}^{(1)} E_j + \varepsilon_0^2 \sum_{jk} \chi_{ijk}^{(2)} E_j E_k + \varepsilon_0^3 \sum_{jkl} \chi_{ijkl}^{(3)} E_j E_k E_l + \cdots.$$

An *isotropic* material has off-diagonal elements equal to zero, diagonal elements that are equal to each other and no spontaneous polarisation P_{0i}. In this special case the above expression reduces to a simple power-series expansion

$$P_i = \varepsilon_0 \chi^{(1)} E_i + \varepsilon_0^2 \chi^{(2)} E_i^2 + \cdots$$

where $\chi^{(n)} = \chi_{11}^{(n)} = \chi_{22}^{(n)} = \chi_{33}^{(n)}$. The terms of order E^2 and above can be neglected for many materials in moderate fields in which circumstances

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$$

where the scalar constant χ is called the *linear dielectric susceptibility* of the material. If this is an adequate approximation then

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1+\chi) \mathbf{E} = \varepsilon \mathbf{E}$$

and ε is the *permittivity* of the dielectric. The abbreviation LIH (linear, isotropic and homogeneous) is often used in this context. The *dielectric constant* κ is defined by

$$\kappa = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi = \varepsilon_{\mathrm{r}} \qquad \text{(the } \textit{relative permittivity}\text{).}$$

All these “constants” are frequency dependent to some extent because it takes a finite time for the dipoles to respond to the electric field.

Anisotropic materials which exhibit a large spontaneous polarisation P_{0i} are known as *ferroelectrics*. A permanently polarised ferroelectric sample is sometimes called an *electret* (there is an analogy with a magnet). $BaTiO_3$ is an excellent example of a ferroelectric material. Related effects include changes in polarisation due to mechanical distortion of this crystal, *piezoelectricity*, and due to temperature changes, *pyroelectricity*.