

Magnetic Constants and Materials

Like dielectrics, magnetic materials can have very complicated properties. A general expression for the components of the *magnetisation* \mathbf{M} at a point where the *magnetic field intensity* is \mathbf{H} is

$$M_i = M_{0i} + \sum_j \chi_{ij}^{(1)} H_j + \sum_{jk} \chi_{ijk}^{(2)} H_j H_k + \sum_{jkl} \chi_{ijkl}^{(3)} H_j H_k H_l + \dots$$

An isotropic material has off-diagonal elements equal to zero, diagonal elements that are equal to each other, and no spontaneous magnetisation M_{0i} . In this special case the above expression reduces to a simple power-series expansion

$$M_i = M_{0i} + \chi^{(1)} H_i + \chi^{(2)} H_i^2 + \dots$$

where $\chi^{(n)} = \chi_{11}^{(n)} = \chi_{22}^{(n)} = \chi_{33}^{(n)}$. The terms of order H^2 and above can be neglected for many materials in moderate fields in which circumstances

$$\mathbf{M} = \chi \mathbf{H}$$

where the scalar constant χ is called the *magnetic susceptibility* of the material. If this is an adequate approximation then

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi) \mathbf{H} = \mu \mathbf{H}$$

and μ is the *permeability* of the material. The abbreviation LIH (linear, isotropic and homogeneous) is often used in this context. The *relative permeability* μ_r is defined by

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi.$$

All these “constants” are frequency dependent to some extent because it takes a finite time for the dipoles to respond to the magnetic field.

Diamagnetism

All atoms have a diamagnetic response to some extent, although the effect will be swamped by the larger paramagnetic or ferromagnetic response if it is present. In classical terms the electrons in filled atomic shells are treated as current loops circling about an axis along the direction of the magnetic field \mathbf{B} . If the orbit radii were not to change as the field increases from zero then the energy of the clockwise loops would decrease and that of the anticlockwise loops would increase. The atom responds by shrinking the radius of the clockwise loop, reducing its moment, and increasing the moment associated with the anticlockwise orbit. The net effect is that the atom acquires a small dipole moment characterised by a negative susceptibility $\chi \sim -10^{-5}$, which is independent of temperature.

The quantum mechanical treatment of the problem is a text-book exercise in perturbation theory using a Hamiltonian that is equivalent to the classical magnetic energy of a current loop of area A , *i.e.*

$$\mathcal{H} \propto (x^2 + y^2)B_z^2$$

and a useful trick to help calculate expectation values is to spot that $\langle x^2 + y^2 \rangle = \frac{2}{3}\langle r^2 \rangle$.

Many pure substances are diamagnetic, *e.g.* argon, silicon, ammonia, Araldite™ and metals with atomic numbers high enough for the diamagnetism of the core electrons to swamp the paramagnetism associated with the band structure.

Paramagnetism

Substances containing ions with non-zero spins are paramagnets and typically have small positive susceptibilities with $\chi \ll 1$. An ion or atom with an incomplete electron shell will have a non-zero net spin and therefore a non-zero magnetic dipole moment. If its total spin quantum number J were very large then an atom would behave as though it had a classical magnetic dipole moment and the susceptibility would be described by the Langevin function, as was the case with paraelectric materials. However, the quantum numbers involved are usually small and a quantum mechanical treatment is necessary. In the common case of a $J=1/2$ spin the calculation reveals that the spin has two eigenstates in an applied field \mathbf{B} and gives the energies and component of magnetic moment along the z -axis (defined by the direction of the applied field) as

$$\begin{aligned} E_{\uparrow} &= -\frac{1}{2}g\mu_B B_z & E_{\downarrow} &= +\frac{1}{2}g\mu_B B_z \\ m_{\uparrow} &= +\frac{1}{2}g\mu_B & m_{\downarrow} &= -\frac{1}{2}g\mu_B \end{aligned}$$

where $g=2$ is the *gyromagnetic ratio* of the electron, and $\mu_B = 9.274 \times 10^{-24} \text{JT}^{-1}$ is the Bohr magneton. The spins are weakly interacting, and hence treated as independent, so the occupation probability of the states has a Boltzmann distribution. From this the mean dipole moment along the z -axis for an ensemble of such spins is found. The outcome is that the magnetisation \mathbf{M} of a sample of number density N atoms each having a total spin J in an applied field \mathbf{B} at temperature T is

$$\mathbf{M} = \hat{\mathbf{B}} N J g \mu_B F_J(J g \mu_B B / k_B T)$$

where the function F_J is called the *Brillouin function* (figure 1) and becomes equal to the Langevin function in the classical limit of large J . If the field is not too large, and the temperature is not too low, a paramagnetic sample can be treated as having a linear susceptibility given, to an accurate approximation, by

$$\chi = \mu_0 \left(\frac{\partial M}{\partial B} \right)_{B=0} = \frac{N \mu_0 g^2 \mu_B^2 J(J+1)}{3k_B T}$$

Experiments on real samples show that there usually are interactions between spins which must be accounted for by a slightly modified expression

$$\chi = \frac{N\mu_0 g^2 \mu_B^2 J(J+1)}{3k_B(T - T_C)} \quad \text{for } T > T_C$$

where T_C is known as the *Curie temperature* and increases in magnitude with the strength of the interactions. The singularity in the susceptibility that occurs at the Curie temperature indicates that the sample undergoes a phase transition at this temperature.

Do not confuse the behaviour described above with the phenomenon known as *Pauli paramagnetism* which arises in some metals as a result of the structure of part filled bands. This can also give rise to small positive values of susceptibility but has a weak temperature dependence.

Ferromagnetism

Below their Curie temperature paramagnetic substances order, *i.e.* the dipoles arrange themselves in regular patterns. The most important of these patterns is known as *ferromagnetic order* in which each dipole tends to point in the same direction as its neighbour. The observed behaviour suggests that there is a strong interaction between adjacent dipoles tending to align them, but it drops off very rapidly with distance and only affects nearest neighbour pairs. This short-range *exchange interaction*, which needs quantum mechanics to explain its details, competes with the long-range dipole-dipole interaction which favours antiparallel arrangements having zero net moment.

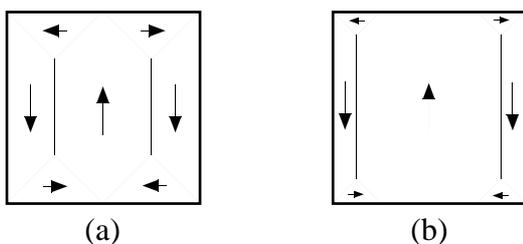


Figure 2. Schematic diagram of a domain structure with (a) $M=0$ (b) $M>0$

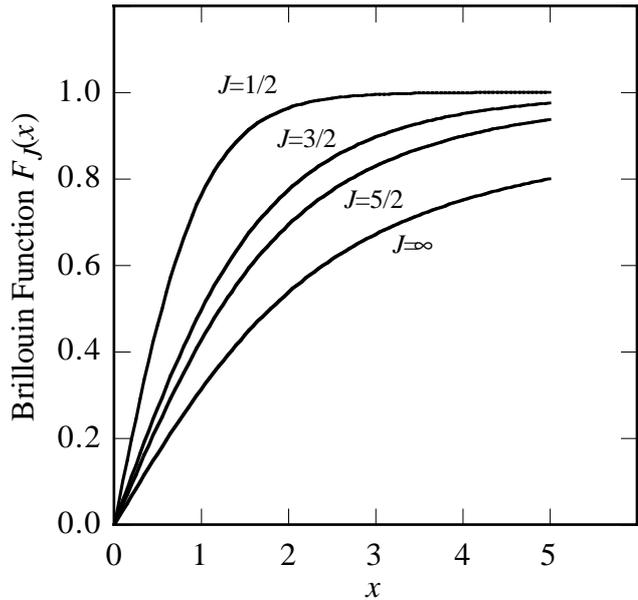


Figure 1. This figure shows the Brillouin function for various values of total spin quantum number J .

The resultant minimum energy arrangement of dipoles is called a *domain structure* (figure 2a) and has no net magnetisation on scales much longer than the size of a single domain, but uniform magnetisation within a domain. The size of these domains varies widely between materials but is usually somewhere in the range $0.1 \mu\text{m} - 0.1 \text{mm}$.

Domain wall movements dominate the magnetic behaviour of ferromagnetic materials. Starting with an initially demagnetised sample with $M=0$ and $B=0$ an

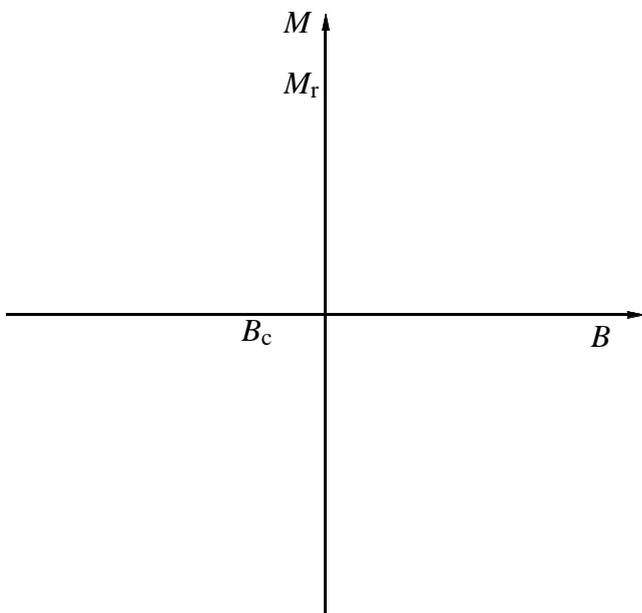


Figure 3. The M - B loop of a ferromagnet.