Quantum Transmission of Atoms Through a Slab of Superfluid Helium

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We describe a measurement of the transmission probability of ^{4}He atoms through a freely suspended slab of superfluid ^{4}He at low temperatures. In our experiment the slab is realised by using an array of parallel cylindrical holes of diameter $51\,\mu$m in a glass disc of thickness $190\,\mu$m. By controlling the chemical potential, the holes can be made to fill or empty with liquid, and the surface curvature varied. We have measured the transmission of atom beams, generated by a thin-film heater and detected with a sensitive bolometer, through this structure. The results show that the dominant transmission channel is atom–R$^+$ roton–atom with a probability $p \sim 0.12$ and that R$^+$ rotons can undergo total internal reflection at the free liquid surfaces.

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Superfluid ^{4}He (helium-II) has a unique combination of properties [1] which make it possible to study the phenomena of quantum evaporation and quantum condensation. At $T \sim 0\,$K helium is an inviscid liquid, with zero vapour pressure, and a free surface that is smooth on an atomic scale. Its excitations are quasiparticles that can travel ballistically and can have energies sufficiently high to eject an atom. When a single excitation in the liquid annihilates at the free surface and ejects a single atom into the vacuum the process is known as *quantum evaporation* [2–4]. This process conserves energy and the momentum-component parallel to the surface [5]. *Quantum condensation* [6, 7] is the inverse process; an atom incident on the surface is adsorbed creating a single excitation in the liquid. Parallel momentum is also conserved in this process, and it is known that an R$^+$ roton (positive group-velocity) is more likely to be created than a high-energy phonon.

The outstanding current goal is to measure the probability of the various types of quantum evaporation and condensation processes. This is difficult to do directly because existing detectors have an unknown and low responsivity to rotons, and hence it is not possible to determine directly the flux of rotons in the liquid. In this letter we describe how we have overcome this obstacle by performing a new type of experiment in which a beam of atoms is directed at one side of a 'slab' of helium, the excitations pass through the slab and a beam of atoms emerges from the other side.

Our helium slab experiment only requires relative measurements of the incident and emitted atom fluxes, so the absolute responsivity of the detector does not need to be known. A parallel slab appears simply to attenuate the flux of atoms. A limitation is that it gives the product of the condensation and evaporation probabilities. However, the transmission of helium atoms through a thin slab of liquid has been treated in recent theoretical calculations [8–10] and the R$^+$ evaporation probability has been estimated in other ways [11, 12].

Quantum evaporation by phonons [3] and rotons [4] is well established [13, 14]. For a roton, the probability of the process does not depend strongly on angle of incidence or energy [5, 14]. There is some indirect evidence that the evaporation probability for R$^+$ rotons is around 0.35 [11, 12]. We also know that the evaporation probabilities of phonons and R$^-$ rotons (negative group-velocity) are one and two orders-of-magnitude smaller respectively [7, 15]. It is likely that other processes occur besides the one-to-one processes. For example, an atom incident on the surface may create multiple ripplons [16, 17]. However, it seems that the simultaneous creation of a free atom and a ripplon has a low probability as there is no angular spreading of the phonon-evaporated atom beam due to the co-creation of low-energy ripplons [14].

Various theoretical studies (refer to Sobnack *et al.* [18] for a survey) of quantum evaporation have predicted probabilities for the one-to-one processes. They generally show a higher probability for for evaporation by R$^+$ rotons than by R$^-$ rotons, but a higher probability for evaporation by high-energy phonons than rotons, and a strongly energy dependent probability, which are not found in experiments [19, 20]. Only one approach [9, 10] has included the possibility of creating ripplons. They analyse atoms impinging on a slab of helium-II and find that the 'transmission' probability is typically ~ 0.1 over much of the energy range above $\sim 10.5\,$K, although low-energy excitations (7.2–9.5 K) have higher probability, ~ 0.25, and there is a deep minimum at 10 K, which corresponds to the position of the roton minimum in their model, where the rotons have zero group-velocity.

In our experiment (figure 1) we created an effective slab of liquid helium by using a thin glass plate with a set of parallel holes perpendicular to the plate. The holes were cylindrical with length $l = 190\,\mu$m and diameter $d = 51 \pm 1\,\mu$m [26]. The curvature of the free surface of liquid in the holes was controlled by changing the amount of helium in the cell. Equating the chemical potentials

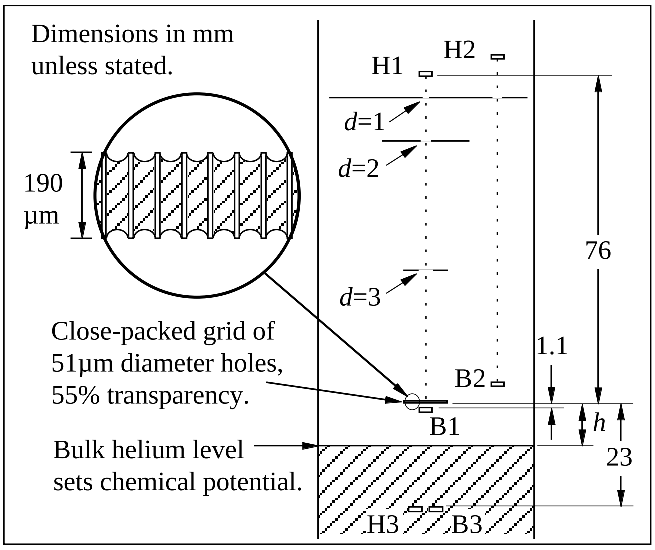

FIG. 1: Schematic diagram of the grid and the cell. The paths of the atom beams are shown by dotted lines.

of the helium in the holes and in the bulk liquid below, the radius of curvature is $R = 2\sigma/\rho g h$ where σ is the surface free energy of the liquid-vacuum interface, ρ is the density of the liquid, and h is the height of the curved surface above the flat surface of bulk liquid. With $\sigma = 3.75 \times 10^{-4}\,\mathrm{J\,m^2}$ [21] and $\rho = 145\,\mathrm{kg\,m^{-3}}$ then $R = 5.27 \times 10^{-7}\,\mathrm{m^2}/h$.

In principle, the system is hysteretic; the energetics dictate that the holes must spontaneously empty when $2R \leq d$ and correspondingly $h \geq 4\sigma/\rho g d$. They will spontaneously fill at a smaller value of h when the bulk level is being raised. In practice, the holes filled when the height difference was $h \sim 20\,\mathrm{mm}$, *i.e.* the lowest point in the bistable region. This was probably due to the cell geometry and filling method. Once filled, the holes remained stable for $h < 20\,\mathrm{mm}$. In our experiment the minimum value of h is 2.3 mm so the maximum value of R is $2.29 \times 10^{-4}\,\mathrm{m}$, which is $4.50d$. At this level the maximum angle between the liquid surface and the horizontal is 6.4° and occurs at the rim of the hole. (In principle, it is possible to use a caesium barrier to separate the reservoir and the plate so that $h = 0$ and the liquid surface is flat [22].)

The atom beam was created by a $5\,\mu\mathrm{s}$ electrical pulse of power 2.0 mW applied to a 1×1 mm thin-film gold heater covered by the saturated helium film. The cell was maintained at $\sim 60\,\mathrm{mK}$ by a dilution refrigerator; at this temperature the vapour pressure is negligible and an atom beam is not scattered by ambient atoms. Three collimators in front of the heater formed the narrow beam of atoms incident on the glass capillary array. The useful angular width of the beam was ±0.8°. The geometry was chosen to ensure that the atoms travelled essentially parallel to the axis of the experiment and perpendicular to the plate. A superconducting zinc-film bolometer was positioned 1.1 mm below the plate. It was biased with a magnetic field to reduce its transition temperature to $\sim 300\,\mathrm{mK}$ and linearised by employing a constant temperature mode of operation [23].

The beam path and the bolometer were laser-aligned. The diameter of the atom beam on arrival at the glass plate was 5 mm and the bolometer area was $1\,\mathrm{mm}^2$. A similar heater–bolometer pair (H2–B2), but without the intervening capillary array, was included to monitor the influence of changing h on the detector due to changes in the thickness of the helium film covering it. These are small, but not negligible, effects – the maximum correction required to the primary signal amplitude is 24% for the smallest h. The intrinsic time constant of the detector was $\sim 1\,\mu\mathrm{s}$, but this does not include the time needed to equilibrate the heat deposited by atoms condensing onto the helium film adjacent to the bolometer. The energy flux detected by the bolometer is interpreted as the atom energy flux convolved with a time constant of $\sim 100\,\mu\mathrm{s}$ arising from this mechanism, and this causes a delay of $\sim 50\,\mu\mathrm{s}$ in the peak of the recorded signal. The liquid level in the cell was measured by sonar; a third heater-bolometer pair, was positioned 22.5 mm below the glass plate. The heater sent a $0.1\,\mu\mathrm{s}$ 20 mW pulse of ultrasound to the free surface where it was reflected back to the bolometer. The height of the liquid surface can be determined to within 0.1 mm from the time of flight and the ultrasound velocity $238.3\,\mathrm{m\,s^{-1}}$.

The empty-hole atom signal (figure 2a) was recorded with the liquid level 28 mm below the holes. The level was then raised in small steps by adding pure ^{4}He [24] to the cell. It can be seen, figure 2c, that filling the holes with helium attenuated the signal by between two and three orders of magnitude. We attribute the rising baseline in this figure to ripplons, created by atoms condensing onto the collimators, arriving at the bolometer. Reducing the curvature of the surface, by decreasing h, increased the signal (figure 2b). The results are summarised in figure 3 which plots the peak height of the detected atom pulses measured at different values of h.

Condensing atoms create, *inter alia,* R^+ rotons which are the excitations that transmit energy through the helium slab. Not all the created R^+ rotons necessarily reach the lower surface of the helium slab; some hit the glass side-wall of the capillary and are dissipated. Furthermore, when $h > 6.5\,\mathrm{mm}$ some of those R^+ rotons that do reach the lower surface undergo total internal reflection and therefore do not cause quantum evaporation (figure 3 inset). The helium surface curvature governs both these effects as a result of the refraction which conserves the parallel momentum component at the surfaces. This is described by $k \sin(\phi) = q \sin(\theta)$ where k and q are the wave vectors of the atom and the R^+ roton respectively, and ϕ and θ are their respective angles to the surface normal of the liquid surface. The critical angle for R^+

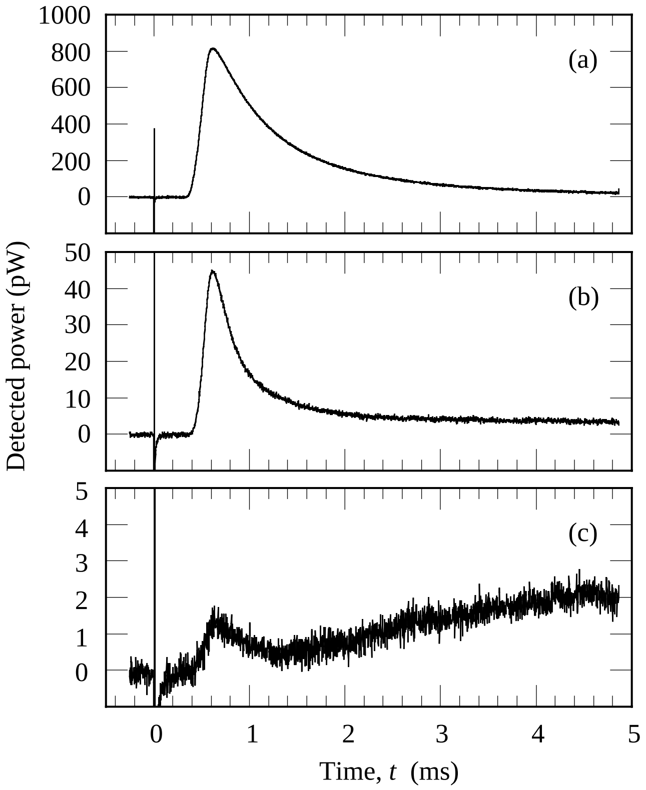

FIG. 2: Typical signals taken at various stages of the experiment: (a) empty holes, (b) liquid-filled holes, $h = 2.3\,\mathrm{mm}$ (minimum surface curvature), (c) $h = 19\,\mathrm{mm}$.

rotons with energy 12.16 K, for example, is 24°.

We have developed a computer model that uses kinematic rules to trace the the atom–roton–atom trajectories and hence calculates the reduction in the evaporated atom flux due to the two loss mechanisms described above. The model assumes that an R^+ roton scatters diffusely from the rough glass wall, probably mode-changing into a R^- roton, and hence is lost. The results for atoms of 4.5 K energy are the curves in figure 3. The points on figure 3 have been corrected, using the H2–B2 measurements, for the changing bolometer responsivity with h, and the scale was chosen to match the measurements to the model. There is agreement between the measurements and the model. The onset of total internal reflection at $h = 6.5\,\mathrm{mm}$ is clearly apparent in both the measurements and the model; this feature is unique to R^+ rotons.

The model provides a basis on which to extrapolate the measurements to $h = 0$ where the surface of the liquid helium would be flat. The ratio of the atom flux extrapolated to $h = 0$ to the atom flux through the empty holes is 0.15. (This value is not significantly affected by the atom energy used in the modelling over the range 4–5 K.) If, instead of the peak heights, we use integrals of the signals over the range $200\,\mu\mathrm{s}$ to $983\,\mu\mathrm{s}$ (this is the time for atoms of energy 1.44 K, which is the minimum required to create a roton, to travel 76 mm) the ratio is 0.12. This is equal to the product of the condensation probability to create R^+ rotons $p_{\mathrm{a}+}$ and the quantum evaporation probability by R^+ rotons $p_{+\mathrm{a}}$, *i.e.* $p_{\mathrm{a}+}p_{+\mathrm{a}} = 0.15$ for an atom energy 4.5 K, and 0.12 on average over the whole energy range.

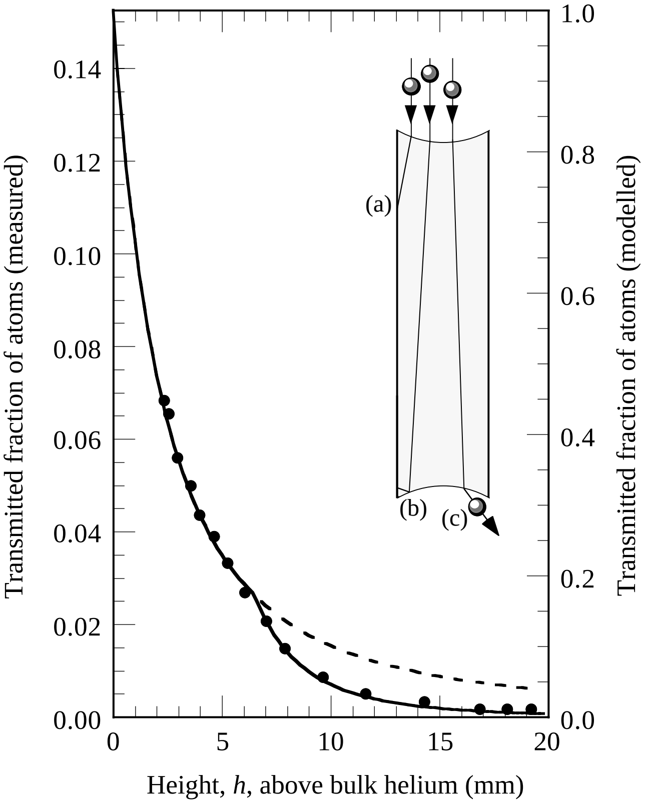

FIG. 3: Fraction of transmitted atoms (points) compared with model (curve) using R^+ roton wave vector $2.2\,\mathrm{\AA}^{-1}$ and assuming that $p_{+\mathrm{a}}p_{\mathrm{a}+} = 1$. The effect of omitting total internal reflection from the model is also shown (chain). The inset illustrates the kinematic model used to interpret the experiment and shows typical events for $h = 10\,\mathrm{mm}$ (a) R^+ scatters at solid surface and is lost, (b) total internal reflection of R^+ at free surface, (c) atom transmitted.

In figure 4 we show the measured atom fluxes through the filled holes, with $h = 2.3\,\mathrm{mm}$, and empty holes as functions of nominal atom energy calculated from time of flight. The presence of the liquid helium attenuates the signal by a factor of 14.8 at the peak. The attenuation is not the same at all atom energies, it is higher on each side of the peak. It might be expected that there should be no signal for atom energies less than 1.4 K when there is helium filling the holes, because such atoms do not have enough energy to create rotons; the energy of the roton minimum is 8.6 K and this must be greater than the sum of the condensation energy, 7.16 K, and the kinetic energy. However, the detector time-constant in our experiment means that the ‘low energy’ signal cannot be identified with a unique atom energy; there is a con-

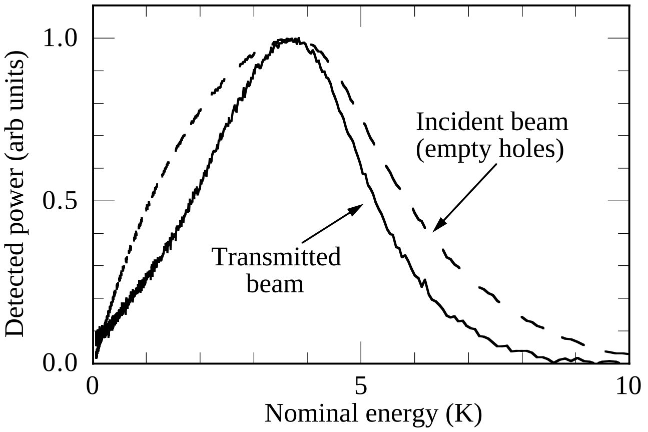

FIG. 4: Signals from figure 2 (a) and (b) rescaled to show the dependence on nominal atom energy $E = 0.5 m_{\mathrm{He}} (D/t)^2$ where D is the heater–detector distance and t is the signal arrival-time.

tribution from higher energy atoms that arrived earlier. There is, however, a significant change in slope around 1.4 K which can be seen in the figure. From the modelling we find the peak atom energy that best fits our data is ~ 4.5 K. The position of the peak in figure 4 is at 3.7 K but the shift to this lower value is accounted for by the detector time-constant, as described above.

In conclusion, we have measured $p_{+\mathrm{a}} p_{\mathrm{a}+}$ the product of the probabilities of one R^+ roton evaporating one atom, and one atom creating one R^+ roton. We circumvented the impasse associated with measuring the R^+ roton flux by measuring the ratio of two atom beams: the beam incident on a freely suspended slab of helium-II and the emergent beam this created. We have shown that the transmission across the liquid slab is mediated by R^+ rotons; these are created by quantum condensation at the top surface and they quantum evaporate atoms at the bottom surface. The average transmission probabilities are remarkably high, $p_{+\mathrm{a}} p_{\mathrm{a}+} = 0.12 \pm 0.01$, as can be appreciated by noting that $\sqrt{0.12} = 0.35$. Dalfovo *et al.* [25] predicted that $p_{+\mathrm{a}} = p_{\mathrm{a}+}$ and we note that if we take $p_{+\mathrm{a}} = 0.35 \pm 0.03$ [11] then our average value implies that $p_{\mathrm{a}+} \sim 0.34$. The measured value of transmission is of similar magnitude to the theoretical values calculated by Campbell *et al.* [9].

The experiment also demonstrates for the first time a number of other interesting properties of R^+ rotons. It confirms that R^+ rotons of energy ~ 12 K can propagate without appreciable attenuation through 0.2 mm of liquid ^{4}He at low temperature. It shows that R^+ rotons undergo total internal reflection when $q \sin(\theta) = k$ at the free surface of liquid helium, and also that R^+ rotons that hit the sidewall are lost from the experiment, presumably because they scatter diffusely and maybe mode-change.

Finally, our measurements show that, not surprisingly, the curvature of the free surface of helium-II in this system is accurately described by conventional thermodynamics, and the emptying transition occurs at the calculated chemical potential.

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