Narrow-Angle Beams of Strongly Interacting Phonons

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We demonstrate that narrow-angle phonon pulses of low energy phonons in liquid ^{4}He are strongly interacting and rapidly come into equilibrium within a narrow cone in momentum space. The effect of collimation on such a system is to strongly reduce the axial phonon flux. This gives a method of separating high and low energy phonons.

PACS numbers: 67.40.-W 67.90.+Z 63.20

1. INTRODUCTION

Phonons in anisotropic beams can exhibit behaviours quite unlike those of isotropic distributions. In its simplest form, an anisotropic distribution of phonons occupies a cone in momentum space so it necessarily has a non-zero average momentum along the cone axis. In real space, this could be a beam, or a propagating pulse, of phonons with an angular width the same as that of the cone in momentum space (fig. 1). An example of the effect of anisotropy is the behaviour of a short phonon pulse injected into liquid ^{4}He at temperatures low enough for ambient thermal phonons to be negligible. Such a pulse is found to break up into two spatially separated groups of phonons[1,2] (fig. 2). The faster group contains low-energy (l) phonons and the slower one contains high-energy (h) phonons.[3,4] The h-phonons are created from the l-phonons by four phonon scattering (4pp)[5] and are lost from the l-phonon group by dispersion. A model[6] for this creation process is based on the theoretical prediction that the l-phonons are in thermodynamic equilibrium. In this paper we present experimental evidence that this is correct.

Isotropic systems of particles, such as atoms or phonons, in thermodynamic equilibrium due to interactions are quite familiar. So also is the behaviour of weakly-interacting particles, such as photons, which can be collimated into narrow-angle beams. In this paper we discuss the less famil-

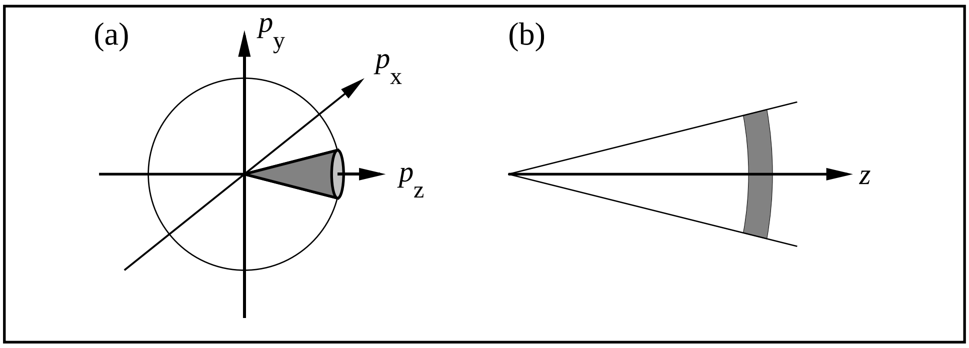

Fig. 1. A simple anisotropic phonon distribution: (a) a cone of points in momentum space, (b) the corresponding real-space picture.

iar behaviour of a narrow-angle beam of particles, namely phonons, which interact strongly and attain equilibrium within the beam.

If a beam of interacting particles is collimated then, after the formation stage, it diverges as it propagates because the particles scatter into unoccupied, but energetically accessible, momentum states. If the particles undergo large-angle scattering then an isotropic distribution is formed on a time-scale of several interaction times. However, the outcome is entirely different if only small-angle scattering processes are allowed. Then, if the cone-angle of the beam is the same or larger than the scattering angles, the cone-angle only slowly increases and the phonons will attain thermodynamic equilibrium with a Bose-Einstein distribution. This notion, that pulses of l-phonons in a cone interacting by three-phonon processes (3pp)[7–9] are in equilibrium,[10] is the basis of the theoretical model which describes the creation of high energy phonons from low energy ones for both short and long pulses, and predicts suprathermal densities of h-phonons.[11,12] This paper reports an experiment that confirms this central idea. It also shows that the h-phonons interact weakly and so can be collimated into a much smaller cone-angle than the l-phonons. This experiment has some important implications, such as the possibility of creating an h-phonon pulse devoid of l-phonons.

2. EXPERIMENTAL METHOD

We investigate the interactions in a phonon pulse by seeing how it develops once it has passed through a collimator. Immediately after passing through the collimator, the pulse has a very narrow angular-width determined by the geometry. As the phonons are interacting, the angular width increases as the pulse propagates away from the collimator. The energy flux

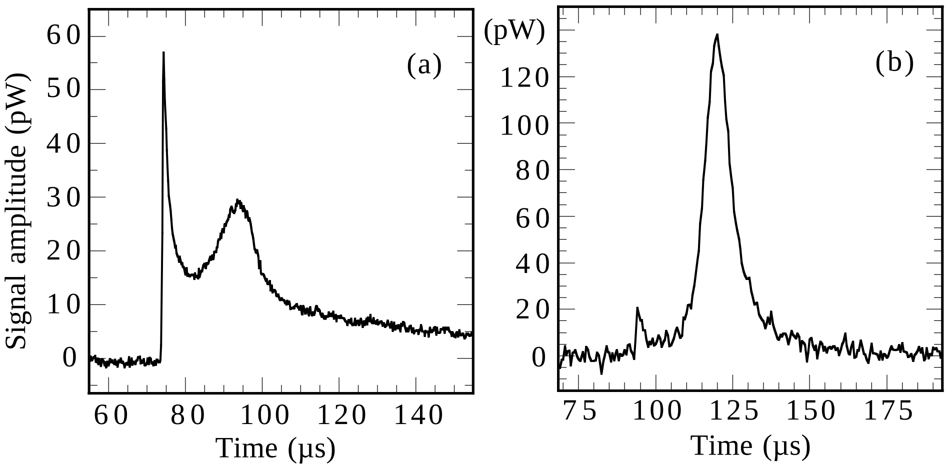

Fig. 2. Measured signals from experiments: (a) without collimation, (b) collimation as shown in fig. 3c.

in the forward direction reduces due to this expansion. This is illustrated in fig. 3 which also shows the experimental arrangement. The source of l-phonons is a $1\,\mathrm{mm}^2$ thin-film gold heater which is electrically heated with power $10\,\mathrm{mW}$ for $0.1\,\mu\mathrm{s}$. The detector is a thin film of superconducting zinc, operated in a constant temperature mode.[13,14] It has a time constant $\tau \approx 1\,\mu\mathrm{s}$ when it is in liquid ^{4}He. The helium in the experimental cell is isotopically pure[15] and is at temperature $\approx 60\,\mathrm{mK}$. Fig. 2b is a typical signal from this experiment, and for comparison fig. 2a is a signal from a previous, uncollimated, experiment. Both experiments detect two pulses arriving as a result of a single input pulse. The faster pulse comprises l-phonons with a typical temperature around $1\,\mathrm{K}$, the slower broader pulse is h-phonons with energies $\geq 10\,\mathrm{K}$. The effect of the collimation can be seen to be very strong; the l-phonon signal is much reduced relative to the h-phonon signal.

3. DISCUSSION

Consider a phonon beam with an angular width less than the 3pp scattering angles. It will rapidly broaden on the time-scale of the phonon-decay lifetime τ_{d}, to a width similar to the 3pp scattering angles. After this initial broadening, the width only increases slowly because creation by the 3pp reduces the angular width of the cone, and within the cone, this exactly balances the decay due to 3pp. Furthermore, the high occupation-factor of the lower momentum states in the cone favour scattering back into the cone through the quantum bracket in the transition probability. The measured[16]

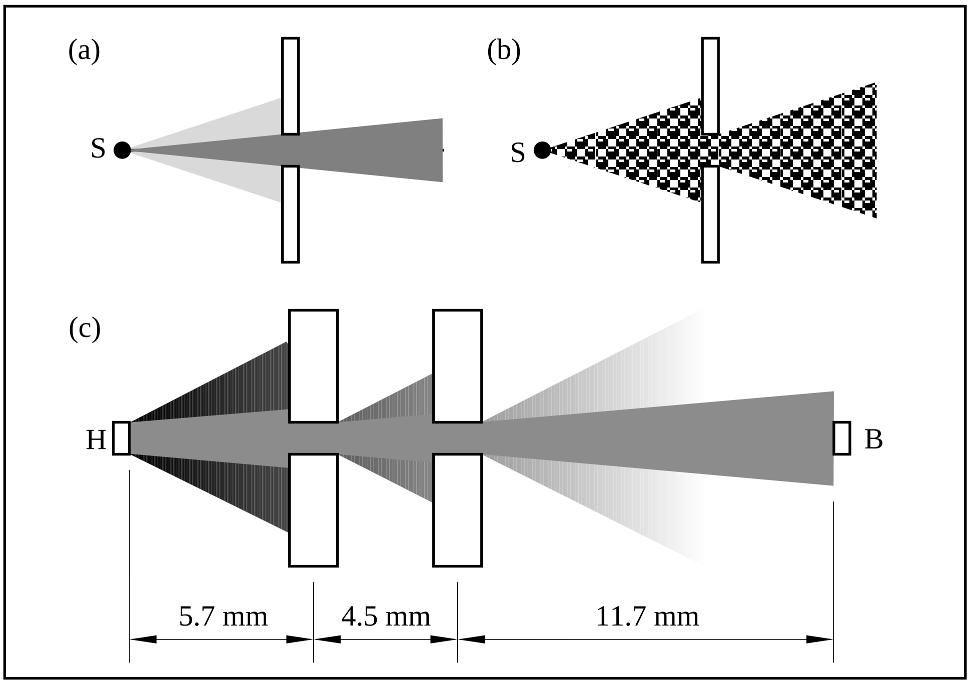

Fig. 3. (a) Effect of collimation on high-energy phonons, and (b) low-energy phonons. (c) The experimental arrangement.

angular half-width at half-maximum of a pulse of l-phonons is $\sim 11°$.

Now consider the development of the l-phonons as they propagate from the heater. The phonons appear to be emitted preferentially in the direction normal to the heater surface, possibly because some are transmitted by the classical acoustic channel. The phonons rapidly thermalise to a temperature T_{p} and form a momentum cone with an angle corresponding to the 3pp scattering angles. The solid angle of the cone in momentum space is Ω, and the thermalised phonons propagate away from the heater essentially preserving this cone angle. The l-phonons cool as they propagate for two reasons. The first is geometric; T_{p} decreases with distance r as $r^{-1/2}$ because the energy density E_{l} decreases as r^{-2} and $E_{\mathrm{l}} \propto T^4$. This is the most important factor at distances greater than the heater dimension. The second cooling-effect is due to 4pp creating h-phonons from the l-phonons by scattering.[6,11] If the pulse length t_{p} is short, *i.e.* $t_{\mathrm{p}} \ll t_{\mathrm{d}}u/c$, where t_{d} is the decay lifetime and u is the difference in velocity between the l- and h-phonons, then the h-phonons are lost from the back of the propagating l-phonon pulse because the h-phonons have a lower group velocity than the l-phonons ($189\,\mathrm{m\,s^{-1}}$ and $238\,\mathrm{m\,s^{-1}}$ respectively).

When $T_\mathrm{p} < 0.7\,\mathrm{K}$ the creation rate of h-phonons is essentially zero[11] and the groups of l- and h-phonons are spatially separated and independent of each other. The h-phonons which are lost by dispersion from the l-phonons are very stable: they cannot spontaneously decay as there are no low-energy phonons to scatter with, and their mutual interaction is negligible.

When the l-phonon pulse arrives at the first collimator, only a small fraction of the energy passes through the aperture. Once through, 3pp broaden the beam again. The pulse temperature again drops as $r_2^{-1/2}$ where r_2 is measured from the aperture in the first collimator. The same thing happens at the second collimator and so the energy density in the l-phonon pulse is very low when it reaches the bolometer (fig. 2b).

We can estimate the expected size of the l-phonon signal if there are strong, but small-angle, interactions between these phonons. If the scattering gives a cone of occupied momentum-states with solid angle Ω, the l-phonons are reduced by the factor

$$R = \frac{\Omega_1 \Omega_2 L^2}{\Omega^2 l_3^2} \tag{1}$$

where Ω_1 and Ω_2 are the solid angles of the collimating holes referred to the heater and to the first collimator respectively, L and l_3 are the distances from the bolometer to the heater and the bolometer to the second collimator, and $\Omega_1, \Omega_2 \ll \Omega$. In our experiment, $L = 21.9\,\mathrm{mm}$, $l_3 = 10.9\,\mathrm{mm}$ and $\Omega_1 = \Omega_2 = 0.026\,\mathrm{srad}$. For $\Omega = 0.124\,\mathrm{srad}$ (corresponding to the measured cone half-angle of $11.4°$) this gives the factor $R \approx 0.18$.

The theory of the interacting l-phonons predicts that a beam of phonons, with 3pp scattering, comes into equilibrium in a time[10], $t_\mathrm{e} \sim 10^{-10}\,\mathrm{s}$. This means that whatever the distribution function of the phonons injected by the heater, within a time t_e they have a Bose-Einstein distribution function characterised by a temperature T_p. This distribution applies up to the critical energy $\epsilon_\mathrm{c}/k_\mathrm{B} = 10.0\,\mathrm{K}$ as phonons with $\epsilon > \epsilon_\mathrm{c}$ can only interact by the much slower 4pp.

The bolometer response depends not only on the incident energy flux, but also on the of the phonon energy spectrum. This is due to the helium–zinc transmission coefficient being frequency dependent.[17] For phonon energies $\epsilon < 5\,\mathrm{K}$ the transmission coefficient depends linearly on phonon energy. The temperature T_p depends on the energy density E_l of the phonons in the cone as $E_\mathrm{l} \propto T^4$. For a reduction factor of 0.18 in the energy flux, the pulse temperature is smaller, by a factor 0.65, than the value without collimation. This reduces the transmission coefficient and hence the bolometer signal by a similar factor. The overall reduction in signal caused by collimation for the above example is $0.18 \times 0.65 \simeq 0.12$. This is consistent with the reduction of the l-phonon signal that we find.

The h-phonons are fully created well before the first collimator.[4] They occupy a $\sim 4^\circ$ cone (HWHM), narrower than the l-phonons.[5,16] Their flux along the axis also decreases as r^{-2}, but is unaffected by the collimation.

4. CONCLUSIONS

We have measured the strong effect that collimation has on l-phonon pulses propagating in cold liquid ^{4}He. The energy flux along the axis of the collimation is much reduced due to the formation of a broadened cone of phonons due to 3pp. This confirms the prediction that the l-phonons are strongly interacting and come rapidly into equilibrium after their distribution in momentum space is radically altered.

ACKNOWLEDGMENTS

We are grateful to Dr M.A.H. Tucker and Dr S. Roshko for help in preparing the experiment, and to EPSRC (GR/N20225) for support.

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