Influence of Backflow on Roton Quantum Evaporation

C.D.H. Williams and M.B. Sobnack*

School of Physics, University of Exeter, Exeter, UK
**Department of Physics, University of Loughborough, Loughborough, UK*

Sobnack et al. recently investigated theoretically the effect of roton backflow on the scattering of atoms, rotons, and phonons at the free surface of superfluid helium-II at $T = 0\,\mathrm{K}$. They treated backflow semi-phenomenologically by modifying the potential in their earlier theory. This paper compares their predictions for the wave-vector dependence of roton quantum-evaporation with time-resolved experiments. It is found that the wave-vector dependence observed in the experiments is much less extreme than was predicted, and we discuss the implications of this result for this type of theory.

PACS numbers: 67.40.Db, 68.45.Da, 67.40.-w

1. INTRODUCTION

The mechanism known as quantum evaporation[1] is a one-to-one process in which an atom is ejected from the free surface of superfluid ^{4}He by the annihilation of a phonon or roton in the liquid. More recently, experiments[2,3] have confirmed that the process conserves both the excitation energy and its wave-vector component parallel to the surface. However, the direct measurement, and theoretical calculation, of evaporation probabilities are still problematic. A bolometric detector in the liquid can measure the flux of high-energy phonons, and it is thought that they have a probability of ~ 0.1 of evaporating atoms.[4] However, the flux of rotons generated by a thin-film heater has defied direct measurement; they do not seem to be detected by bolometers. Attempts to deduce the absolute (R^+−atom) roton evaporation probabilities by indirect means[5–8] seem to suggest a value, typically ~ 0.3, that increases with wave-vector.

The theoretical description of quantum evaporation is also notoriously difficult and, in spite of recent progress, still seems to be an unsolved prob-

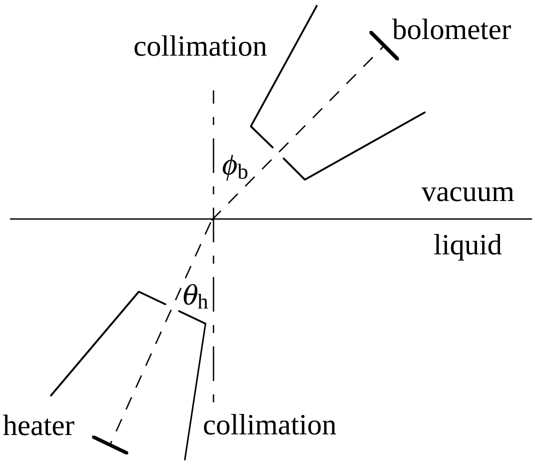

Fig. 1. Schematic diagram of the BW quantum evaporation experiment.

lem. In this paper, recent theoretical calculations[9] of evaporation probabilities are confronted with the results of experiments.[10,11] The comparison is made by installing the calculated evaporation probabilities into a recently developed high-precision simulation[12] of the experiments.

2. EXPERIMENTS

In a comprehensive series of experiments Brown and Wyatt[10,11] (BW) created excitations in the liquid helium by pulsing a thin-film gold heater. The excitations travelled ballistically 6.5 mm to the surface, and there caused evaporation of atoms which, in turn, travelled another 6.5 mm before condensing onto a superconducting-transition bolometer operated in constant temperature mode (see fig. 1). BW recorded time-resolved atom-flux signals for a variety of heater powers, angles of incidence, and detector positions. These confirmed that an excitation in the liquid, with wave vector q at angle θ_h to the surface normal, could evaporate a single atom, with wave vector k at angle ϕ_b, subject to the boundary conditions

$$E(q) - E_b = \frac{\hbar^2 k^2}{2m} \quad \text{and} \quad q\sin(\theta_h) = k\sin(\phi_b) \qquad (1,2)$$

where $E(q)$ is the ^{4}He excitation spectrum and $E_b/k_B = 7.15\,\mathrm{K}$ is the binding energy of an atom, mass m, to the liquid surface at $T = 0\,\mathrm{K}$.

The BW experiment has been re-examined since its original publication; it has been possible to make several minor systematic corrections[12] and the results confirm that the above boundary conditions are accurately obeyed.

3. THEORY

Over the years, there have been several theoretical studies[13] of quantum evaporation. Recently, Sobnack *et al.*[14–16] adapted Beliaev's theory[17] to the inhomogeneous ^{4}He–vacuum system at $T = 0\,\mathrm{K}$ and calculated probabilities for the one-to-one quasiparticle-scattering processes as a function of energy. Although the calculated probability of phonon–atom evaporation was consistent with experimental estimates, the calculated probabilities for the R^+–atom process were too small for low-energy rotons.[15,16,18]

In an attempt to improve their theory, Sobnack *et al.*[9,19] have incorporated a roton-backflow[20] mean-field into Beliaev's theory using techniques employed by polarisation potential (PP) theory.[21] The additional contribution appears as a renormalised single-particle effective mass m^*, and the strength of the backflow potential is proportional to $\Delta m = m^* - m$.

Sobnack *et al.*[9] assumed that the multi-phonon contributions in the PP theory do not affect the quantum evaporation process, and that the effective mass is independent of wave-vector. They found that, in the Bogoliubov limit[22], including the PP backflow is equivalent[23,24] to replacing the effective He-He potential $V(\mathbf{k})$ by $V'(\mathbf{k}) = V(\mathbf{k}) + \hbar^2\omega^2 W(\mathbf{k})$, where $W(\mathbf{k}) = \Delta m/\hbar^2\mathbf{k}^2$. The single-particle Green functions of the superfluid system then have poles at $\hbar\omega = \pm E_\mathrm{B}$, where E_B is the "new" Bogoliubov spectrum[22,23]

$$E_\mathrm{B}(\mathbf{k}) = \left[\frac{\hbar^4\mathbf{k}^4}{4mm^*} + 2\rho_0\frac{\hbar^2\mathbf{k}^2}{2m}V'(\mathbf{k})\right]^{1/2} \tag{3}$$

and ρ_0 is the condensate density. Inclusion of the backflow potential is equivalent to replacing the factor m^2 in the denominator of the first term on the right hand side of eqn 2 of ref. 15 by the product mm^*. The Bogoliubov spectrum, with the choice $V_0 = 15.2\,\mathrm{K}\,\mathrm{\AA}^{-1}$ and $a_0 = 2.1\,\mathrm{\AA}$ for the effective Brueckner potential[25]

$$V(k) = a_0V_0\frac{\sin a_0k}{a_0k}, \tag{4}$$

together with $m^* = 1.4m$, gives a good fit to the measured excitation spectrum of ^{4}He.[23] It was also assumed that all the quasiparticles travel ballistically and have long decay lengths compared with the surface length-scale, and neglecting inelastic (multi-phonon, ripplons) processes. New equations of motion were then derived and solved numerically to find wave functions, and hence the current associated with each quasiparticle or atom. From these currents, the probabilities P_{ij} (probability of state i scattering into state j) of all the one-to-one surface scattering processes allowed by the conservation laws. P_{ij} were calculated for a wide range of energies for oblique incidence (both for fixed parallel momenta and fixed angles of incidence).

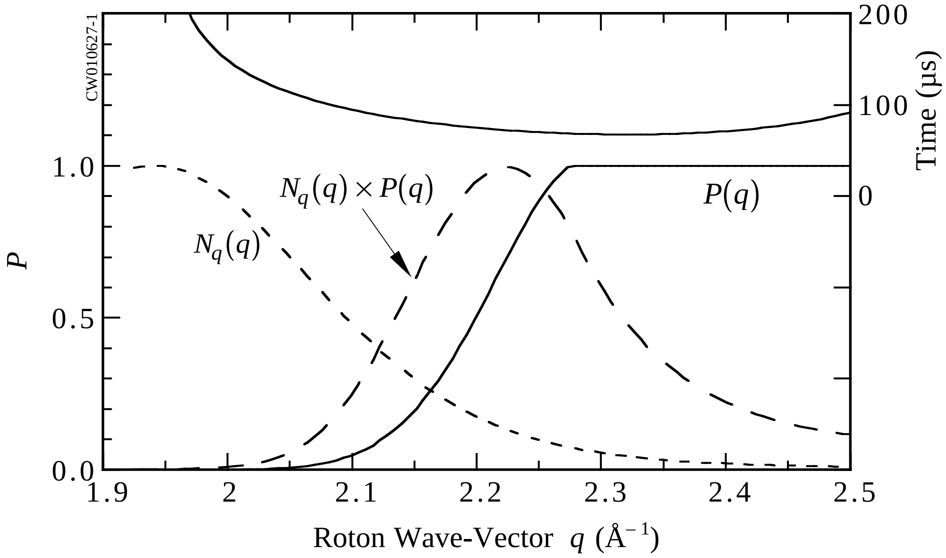

Fig. 2. Illustration of the effect of the theoretical evaporation probabilities described in the text on the roton spectrum $N_{\mathrm{q}}(T_{\mathrm{eff}} = 1.5\,\mathrm{K})$. The corresponding atom arrival times at the detector are also shown.

In fig. 2 we give the R^+ roton evaporation probability P_{+a} for R^+ rotons incident at an angle $\theta_{\mathrm{in}} = 14^\circ$ to the surface normal.

4. SIMULATIONS

The Monte-Carlo simulation used to interpret the BW experiments is a descendent of code originally written by Brown.[11] It creates excitations at points on the thin-film heater surface and tracks their paths through the experiment. Each roton that successfully passes through the collimation, and then evaporates an atom that strikes the bolometer, contributes to the signal; an atom landing on the bolometer adds an appropriate amount of energy to a bin matching its arrival time. The resulting signal is convolved with functions representing the finite heater-pulse duration and the detector time-constant. The evaporation probability is included in the simulation by using it to weight the energy deposited by the condensing atom. As remarked earlier, the injected spectrum of rotons is unknown so the simulation assumes it is of the form

$$N_{\mathrm{q}}(q)\,\mathrm{d}q \propto \frac{q^{\lambda}\,\mathrm{d}q}{\exp(E(q)/T_{\mathrm{eff}}) - 1} \quad \text{where} \quad \lambda = 2 \tag{5}$$

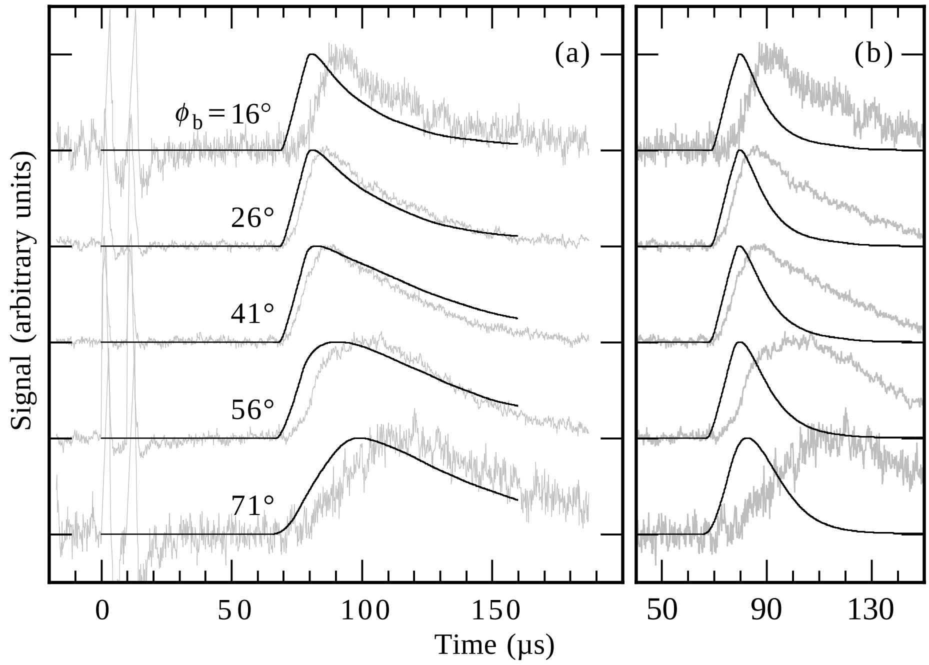

Fig. 3. Comparison of the corrected BW measurements[12] for $\theta_h = 14°$ with simulations using (a) $P(q) = 1$, (b) $P(q)$ as shown in fig. 2.

and $E(q)$ is the low-temperature excitation spectrum of helium-II. The shape of this distribution is dominated by the value of the parameter $T_{\rm eff}$ and is insensitive to the value of the density-of-states parameter over the physically meaningful range $1 \leq \lambda \leq 3$. The value of $T_{\rm eff}$ is selected to fit the time-of-flight measurements; it depends on the heater power and is typically $1.0 -$ -1.5 K, comparable with the transient temperature of the gold film.

5. DISCUSSION AND CONCLUSIONS

The simulation with $P = 1$ (fig. 3a, see also ref. 12) reproduces the experiments quite well. The excitation spectrum used in the simulation[11] is known to under-estimate the measured group-velocities slightly; if this were to be compensated for, the simulations would slightly over-estimate the low-energy roton signal arriving after the peak.

However, the simulations using the calculated probabilities (fig. 3b) do not agree satisfactorily with experiment. The theory clearly attenuates lower-energy rotons too much relative to the higher energy rotons. No physically reasonable modification to the injected spectrum affects this result because of the strong q-dependence of the probabilities calculated by Sobnack *et al.* so it is almost certain that these are to blame. In order to agree with these experiments, the R^+−atom evaporation probability can increase only by about 50% over the range of roton wave-vectors greater than *ca* $2\,\text{Å}^{-1}$, and it will be necessary re-examine the theory in the light of this.

ACKNOWLEDGEMENTS

We gratefully acknowledge discussions with A.F.G. Wyatt, and financial support from the Hong Kong Research Grant Council, China (Project No. HKUST6080/98P) (MBS), and EPSRC GR/N20225 (CDHW).

REFERENCES

1. P.W. Anderson, *Phys. Lett.* **29A** 563 (1969).
2. M.J. Baird, F.R. Hope and A.F.G. Wyatt, *Nature* **304** 325 (1983).
3. F.R. Hope, M.J. Baird and A.F.G. Wyatt, *Phys. Rev. Lett.* **52** 1528 (1984).
4. M.A.H. Tucker and A.F.G. Wyatt, *Czech. J. Physics* **46** 263 (1996).
5. C. Enss, S.R. Bandler, R.E. Lanou, H.J. Maris, T. More, F.S. Porter and G.M. Seidel, *Physica B* **194** 515 (1994).
6. P. Fozooni, D.S. Spencer and M. Lea, *Japan J Appl. Physics* **26-3** 281 (1987).
7. A.C. Forbes and A.F.G. Wyatt, *J. Low Temp. Physics* **101** 537 (1995).
8. M.A.H. Tucker and A.F.G. Wyatt, *J. Low Temp. Physics* **121** 333 (2000).
9. M.B. Sobnack, J.R. Matthias, J.C.H. Fung, C.D.H. Williams, and J.C. Inkson, submitted to *Phys. Rev. B* (2001).
10. M. Brown and A.F.G. Wyatt, *J. Phys. Condens. Matter* **2** 5025 (1990).
11. M. Brown, *Ph.D. Thesis*, University of Exeter (1990).
12. C.D.H. Williams, *J. Low Temp. Physics* **113(3-4)** 627 (1998).
13. References 2–10 of ref. 19 below.
14. M.B. Sobnack and J.C. Inkson, *Phys. Rev. B* **56** R14271 (1997).
15. M.B. Sobnack, J.C. Inkson, and J.C.H. Fung, *Phys. Rev. B* **60** 3465 (1999).
16. M.B. Sobnack and J.C. Inkson, *Phys. Rev. Lett.* **82** 3657 (1999).
17. S.T. Beliaev, *Sov. Physics JETP* **7** 289 (1958).
18. C.D.H. Williams, *J. Low Temp. Physics* **113(1-2)** 11 (1998).
19. M.B. Sobnack, *Phys. Rev. B* **62** 11355 (2000).
20. R.P. Feynman and M. Cohen, *Phys. Rev.* **102** 1189 (1956).
21. C.H. Aldrich and D. Pines, *J. Low. Temp. Physics* **25** 677 (1976).
22. N.N. Bogoliubov, *J. Physics (USSR)* **11** 23 (1947).
23. J.R. Matthias, Ph.D. Thesis, University of Exeter (1998).
24. W.C. Wu and A. Griffin (*unpublished*).
25. K.A. Brueckner and K. Sawada, *Phys. Rev.* **106** 1117 & 1128 (1957).