these pairs of levels during the initial period of time after the radiation is turned on instantaneously.

The fastest process which occurs in the system is the 2s–2p vacancy transition, which results from a Coster–Kroning process, and is characterized in this case by a time $(7\text{-}9)\cdot 10^{-17}$ s (Refs. 1 and 2). If the appearance of an M vacancy[1,2] does not cause a change in the energy difference between the 2p and 1s levels of magnitude greater than the width of the emission line, then the 2p vacancies formed in this manner must be taken into consideration in an analysis of the inversion question for times of 10^{-16}-10^{-15} s in this (Ref. 8). A problem of this type arises in an analysis of 1s–2p Auger transitions of vacancies.[6-8] The maximum possible role which could played by the Coster–Kroning process can be evaluated under the assumption that an L_1 vacancy goes instantaneously to the L_2 and L_3 levels, with respective probabilities $f_{1,2}$ and $f_{1,3}$ (Refs. 1 and 2; a similar assumption was made for sulfur in Ref. 8). The rate of the photoionization production of 2p vacancies is then given by

$$\nu'_{L_2(3)} = \nu_{L_2(3)} + f_{1,2(3)}\,\nu_{L_1}, \tag{5}$$

where $f_{1,2} \approx 0.3$ and $f_{1,3} \approx 0.6\text{-}0.7$ (Refs. 1 and 2). It can be seen from Table I that the values of $\nu'L_2(3)$ differ from $\nu L_2(3)$ by about 30%, and the possibility of satisfying condition (3) for t on the order of 10^{-16} s. remains: ν_K is larger than ν'_{L_2} or $0.5\ \nu'_{L_3}$.

We would thus expect that the application of radiation with $T \approx 10$ keV to an atom with $Z \approx 20\text{-}30$ would result in the creation of a population inversion between the 2p and 1s levels, and there would be an induced amplification of the corresponding emission, although an analysis of the induced amplification requires a more rigorous determination of the vacancy concentration.[5-9]

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Scattering of two-dimensional particles by a short-range potential

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Calculations of the mobility of charge carriers in two-dimensional systems generally use the Born approximation; i.e., the scattering potential is treated as a perturbation. That approach is not always permissible, however.

In this letter we report the results of an exact solution, by the phase-shift method, of the problem of the scattering of a two-dimensional particle by a short-range potential. We find a condition for the applicability of the Born approximation for slow two-dimensional particles and examine resonant scattering in the two-dimensional case. The expressions derived here are used to calculate the mobility of electrons in a square quantum well.

Let us consider the scattering of two-dimensional particles by an axisymmetric potential $U(\rho)$, which is nonzero in only a bounded region. The symmetry axis runs perpendicular to the plane of the motion of the two-dimensional particles. Since the potential is symmetric, we can separate variables in the expression for the wave function:

$$\Psi(\rho,\varphi)=\sum_{m=-\infty}^{+\infty} R_m(\rho)e^{im\varphi}. \tag{1}$$

Outside the range of the scattering potential, the radial function satisfies the free Bessel equation, whose general solution is

$$R_m(\rho)=a_m\left[\cos\delta_m J_m(k\rho)-\sin\delta_m N_m(k\rho)\right], \tag{2}$$

where δ_m is the scattering phase shift, which characterizes the addition of a second, linearly independent solution of the free equation — the Neumann function $N_m(k\rho)$ — to the first — the Bessel function $J_m(k\rho)$.

A simple relationship between the effective transport cross section and the scattering phase shifts was derived for the two-dimensional case in Ref. 1:

$$\sigma=\frac{4}{k}\sum_{m=0}^{+\infty}\sin^2(\delta_m-\delta_{m+1}). \tag{3}$$

The case of primary interest is the scattering of slow particles, specifically, the case $ka \ll 1$, where a is the effective radius of the field $U(\rho)$, and the particle energy $E=\hbar^2k^2/2m^*$ is small in comparison with $|U(\rho)|$ within this radius. Scattering cross section (3) is then dominated by the phase shift δ_0 (s-wave scattering), and the transport scattering cross section is

$$\sigma=\frac{4}{k}\sin^2\delta_0. \tag{4}$$

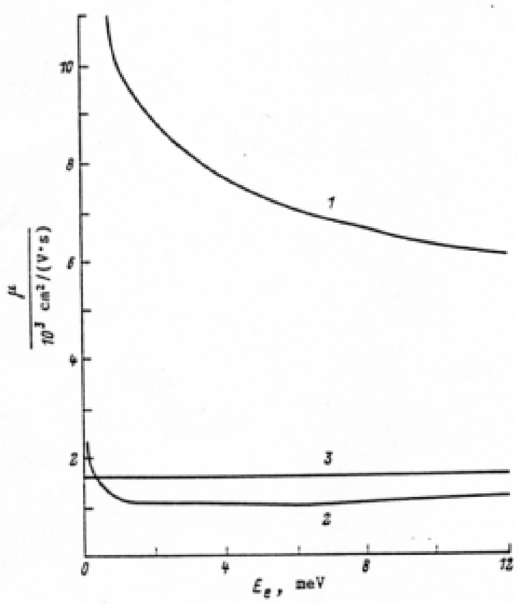

FIG. 1. Mobility due to scattering of electrons in a square quantum well by the roughness of the well boundaries versus the energy of the electrons. 1) $\Delta d < 0$; $\Delta d > 0$; 3) Born-approximation result.

For the scattering of slow particles by a circular potential, $U(\rho) = U_0$ for $\rho \le a$ and $U(\rho) = 0$ for $\rho > a$, we find the following result by joining the logarithmic derivative of the wave function at $\rho = a$:

$$\operatorname{tg}\delta_0 = \frac{-\pi/2}{\ln\frac{2e^{-c}}{ka} - \frac{J_0(k_0a)}{k_0a J_1(k_0a)}}, \quad U_0 < 0; \tag{5}$$

$$\operatorname{tg}\delta_0 = \frac{-\pi/2}{\ln\frac{2e^{-c}}{ka} - \frac{I_0(k_0a)}{k_0a I_1(k_0a)}}, \quad U_0 > 0 \tag{6}$$

Here $k_0 = \sqrt{2m^*|U_0|}/\hbar$, and c is Euler's constant.

The scattering cross section calculated from (4)-(6) is the same as the perturbation-theory result $\sigma = \frac{\pi^2}{k}\left(\frac{m^* U_0}{\hbar^2} a^2\right)^2$ when

$$|U_0| \ll \frac{\hbar^2}{m^* a^2} \cdot \frac{1}{\ln\frac{2e^{-c}}{ka}}. \tag{7}$$

The condition for the applicability of the Born approximation for slow particles in the two-dimensional case, (7), differs from the known three-dimensional condition[2] by a factor $\ln^{-1}(2e^{-c}/ka)$. In other words, the condition is more stringent in this case and contains a dependence on the energy of the particle being scattered.

In the opposite limit (scattering by an impenetrable circle, $k_0 a \gg 1$), expressions (4) and (6) lead to

$$\sigma = \frac{4}{k} \frac{1}{1 + \frac{4}{\pi^2}\ln^2\left(\frac{2e^{-c}}{ka}\right)}. \tag{8}$$

It can be shown that for an arbitrary short-range repulsive potential the cross section for the scattering of slow two-dimensional particles is again given by expression (8), where a is the effective radius of the potential. As the wave vector of the particle being scattered approaches zero, scattering cross section (8) increases without bound, instead of approaching a finite limit ($\sigma = 4\pi a^2$) as in the three-dimensional case.

A case of particular interest is that of scattering by a potential well which contains a shallow s level, i.e., the case of the resonant scattering of two-dimensional particles. The scattering cross section in this case depends on the ratio of the energy of the two-dimensional particle, E, to the energy of the shallow level, ε:

$$\sigma = \frac{4}{k} \frac{1}{1 + \frac{1}{\pi^2}\ln^2\frac{E}{|\varepsilon|}}. \tag{9}$$

The expression can be derived without difficulty from (4), (5) and from the expression for the energy of a shallow level in a square two-dimensional well, but it is valid for an arbitrary potential well with a shallow level. Although expression (9) is quite different from the three-dimensional result for the cross section for resonant scattering,[2] the expression for σ in the two-dimensional case has a pole at $E = -|\varepsilon|$, as in the three-dimensional case.

These results can be used to calculate the mobility in a specific two-dimensional system consisting of a square quantum well which is formed by a narrow GaAs layer in a (GaAl) As solid solution. Figure 1 shows the calculated mobility due to scattering by rough surfaces of a quantum-well layer as a function of the energy of the charge carriers. We used the following values for the parameters of the structure in these calculations: layer thickness d = 30 Å, roughness height $\Delta d = \pm 2.8$ Å, roughness radius a = 20 Å, and roughness surface concentration $N_{SR} = 5\cdot10^{11}$ cm^{-2}. The upper curve in Fig. 1 corresponds to the scattering by a roughness which marrows the quantum-well layer, i.e., to scattering by potential hills; the lower curve corresponds to scattering by a roughness which widens the layer (a scattering by potential valleys); and the straight line shows the result of the Born-approximation calculation.

The nature of the energy dependence of the mobility determines the shape of the current—voltage characteristic under heating conditions. According to the Born approximation, heating does not affect the mobility. A phase-shift calculation shows that the mobility should decrease (the current—voltage characteristic should be sublinear) as a result of scattering by potential hills.

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