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Spin waves in a magnonic crystal with sine-like interfaces

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Abstract

The spectrum of exchange spin waves in a magnonic crystal (periodic magnetic multilayer) with diffuse interfaces is derived for a model with a cosine-like profile of the uniaxial anisotropy value at the interfaces. The dependence of the band gap size upon the interface thickness and the depth of modulation of the anisotropy value is analyzed. In particular, it is shown that diffuse interfaces may lead to a magnonic spectrum in which band gaps have size equal to or even larger than those in the model with sharp (infinitely thin) interfaces (Kronnig–Penney model).

Keywords: Magnonics; Magnon; Superlattice; Nanomagnetism

Due to the wealth results of investigations of photonic crystals [1] and other similar objects with artificially created periodicity [2–6], periodic magnetic structures have received a renewed attention. Such structures could be used to control the propagation of spin waves (SW) [7,8]. By analogy to photonic crystals, periodic magnetic media are referred to as *magnonic crystals* (MCs). The knowledge of the SW spectrum of magneto–photonic crystals (dielectric MCs that support magnetic field controlled photonic band gaps [9]), is also required if they are to be used within devices operated at a GHz clock rate.

While most of the theoretical efforts have been devoted to investigation of MCs with infinitely thin interfaces [10–14], interfaces of realistic samples always have a finite thickness. For example, resonant X-ray magnetic scattering analyses show that the magnetic interface can be thicker than the corresponding chemical interface [15]. On the basis of the full Ginzburg–Landau free energy functional, Schwenk et al. [16] considered the effect of finite temperatures upon both statics and dynamics of magnetization in an all-ferromagnetic periodic multilayer. In particular, they showed that, even when multilayer interfaces are chemically sharp, the static magnetization magnitude profile is always smooth so that magnetic "transition" layers are formed between the basic constituent layers. In addition, the interfaces can contain some roughness, which was predicted to significantly modify SW spectrum in the case of thin magnetic films [17,18]. Therefore, a question arises whether the realistic multilayers can efficiently act as MCs.

Gorobets et al. [19] used a harmonic function to approximate the coordinate dependence of magnetic parameters of a MC with diffuse interfaces. A similar model was used for the investigation of a planar metallic MC by Kolodin and Hillebrands [20]. Ignatchenko et al. [21,22] used a model based upon the Jacobian elliptic sine function, which can describe interfaces of arbitrary thickness, to demonstrate a strong dependence of the magnonic spectrum and coefficients of reflection and transmission of SWs upon the thickness of interfaces. Although being more realistic than that of Gorobets et al., the model of Ignatchenko et al does not allow one to obtain an exact solution, and so is based upon the perturbation theory in the limit of small modulation of the parameters of the MC. At the same time, the theory of waves in inhomogeneous media is well developed [1,2,4], and can be used to build an exactly solvable model for investigation of SWs in a MC in which both the thickness

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of interfaces and the amplitude of modulation of the MC parameters have finite (maybe quite large) values [23]. In the present work, such a model is developed for a one-dimensional MC in which the distribution of the value of the uniaxial anisotropy constant at the interfaces is approximated with a cosine function, while the anisotropy value is assumed to be constant within the basic constituent layers of the MC.

Let us consider an infinite one dimensional MC represented by a system of alternating uniform magnetic layers of two different types but of the same thickness d. The layers have different values β_- and β_+ of the uniaxial anisotropy constant β, while the values of the exchange interaction parameter α, gyromagnetic ratio g, and the saturation magnetization M_0 are assumed to be constant throughout the sample. We assume that the "basic" layers of the MC are separated by "transition" regions of thickness δ in which the value of the uniaxial anisotropy constant changes continuously between values β_- and β_+. The OZ axis is chosen to be perpendicular to the plane of the layers. The easy axis (EA) and the internal magnetic field H both lie parallel to the OZ axis. n is the unit vector in the direction of the internal magnetic field H. Here, we neglect magnetic damping. The effects associated with its presence were considered in Refs. [19,20,24–26]. In particular, the case of an interface dominated SW damping was discussed in Refs. [25,26].

To describe the dynamics of the magnetization $\mathbf{M}(\mathbf{r},t)$, we use the Landau–Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t} = -g\left[\mathbf{M} \times \left\{(H + \beta(\mathbf{Mn}))\mathbf{n} + \frac{\partial}{\partial \mathbf{r}}\left(\alpha \frac{\partial \mathbf{M}}{\partial \mathbf{r}}\right)\right\}\right]. \quad (1)$$

For the present geometry, the dynamic magneto-dipole field is absent [7]. Magnonic crystals properties of which are dominated by the magneto-dipole interaction are studied, for example, in Ref. [8].

Let us consider small deviations $\mathbf{m}_j (j = 1\ldots4)$ of the magnetization of each layer from the ground state given by a uniform magnetization parallel to the EA

$$\mathbf{M}_j(\mathbf{r},t) = \mathbf{n}M_0 + \mathbf{m}_j(\mathbf{r},t), \left|\mathbf{m}_j\right| \ll M_0. \quad (2)$$

Linearizing Eq. (1) with respect to $\mathbf{m}_j$, and introducing Fourier components $\mathbf{m}_j(\mathbf{r},t) = \mathbf{m}_j(\mathbf{r})\ \exp\{i\omega t\}$ and then variable $\mu = m_x + im_y$, we obtain the following equation

$$\frac{d^2\mu(z)}{dz^2} + k^2(z)\mu(z) = 0, \quad (3)$$

where $k(z) = \sqrt{(\Omega - h - \beta(z))/\alpha}$, $\Omega = \omega/gM_0$, and $h = H/M_0$.

We have to find such a function $\beta(z)$ that function $k(z)$ is periodic with period L, is continuous, and has continuous derivatives. In addition, function $\beta(z)$ (and hence $k(z)$) must be constant within the main layers of the MC, and it must be possible to find an analytical solution of equation (3) in the transition layers. These conditions are satisfied by the following function

$$\beta(z) = \begin{cases} \beta_1 = \beta_- = \langle\beta\rangle - \frac{\Delta\beta}{2} & z_0 + nL < z < z_1 + nL, \\ \beta_2 = \langle\beta\rangle + \frac{\Delta\beta}{2}\cos\left[\frac{\pi}{\delta}(z - z_2)\right] & z_1 + nL < z < z_2 + nL, \\ \beta_3 = \beta_+ = \langle\beta\rangle + \frac{\Delta\beta}{2} & z_2 + nL < z < z_3 + nL, \\ \beta_4 = \langle\beta\rangle - \frac{\Delta\beta}{2}\cos\left[\frac{\pi}{\delta}(z - z_4)\right] & z_3 + nL < z < z_4 + nL, \end{cases} \quad (4)$$

where $\langle\beta\rangle = (\beta_+ + \beta_-)/2$, $\Delta\beta = \beta_+ - \beta_-$. $z_j (j = 1\ldots4)$ are the coordinates of the boundaries between the basic and transition layers ($z_0 = 0$, $z_1 = d$, $z_2 = d + \delta$, $z_3 = 2d + \delta$, $z_4 = 2d + 2\delta = L$), $n = 0, \pm1, \pm2, \ldots$, and L is the period of the MC. The graph of function $\beta(z)$ is shown in Fig. 1.

At the interfaces z_j, the solution of Eq. (3) must satisfy the boundary conditions that in the exchange limit [12] are

$$\mu_j\big|_{z_j} = \mu_{j+1}\big|_{z_j}, \left.\frac{\partial \mu_j}{\partial z}\right|_{z_j} = \left.\frac{\partial \mu_{j+1}}{\partial z}\right|_{z_j}, j = 1\ldots4. \quad (5)$$

The physical meaning of the first of them is that, in the exchange-dominated regime, the directions of the magnetization on both sides of the interfaces are parallel. The second condition ensures that the normal to the interface component of the energy flux density vector is continuous [12].

Furthermore, the solution of Eq. (3) must satisfy the periodicity condition, i.e., the dynamic magnetization at the boundaries of the period $z = z_0 = 0$ and $z = z_4 = L$ can differ only by a phase factor [2]

$$\mu(0) = \exp\{iKL\}\mu(L), \quad (6)$$

where K is the Bloch wave number.

Let us find solution $\mu_j(z) = \mu(z_{j-1} < z < z_j)$ for each layer. In the basic layers (for $z_0 < z < z_1$ and for $z_2 < z < z_3$), the solution of Eq. (3) has the form of the plane waves

$$\mu_j(z) = \mu_j^+ \exp\{+ik_jz\} + \mu_j^- \exp\{-ik_jz\}, j = 1, 3, \quad (7)$$

where μ_j^+ and μ_j^- are the wave amplitudes. In the transition layers (for $z_1 < z < z_2$ and for $z_3 < z < z_4$), Eq. (3)

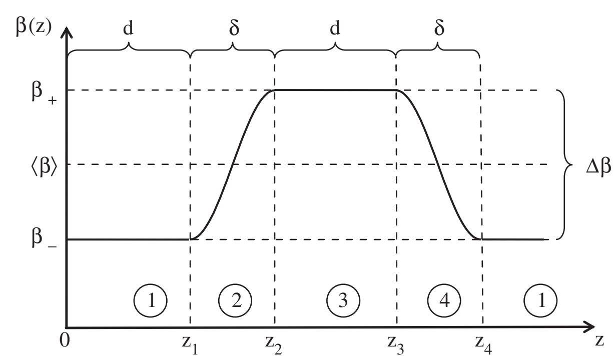

Fig. 1. The coordinate dependence of the anisotropy constant is shown for one period MC, consisting of an infinite number of such periods, i.e., …–1–2–3–4–1–2–3–4–…

transforms into

$$\frac{d^2\mu_j(z)}{dz^2}+\left[\frac{\Omega-\langle\beta\rangle-h}{\alpha}+(-1)^{j/2}\frac{\Delta\beta}{\alpha}\cos\left(\frac{\pi}{\delta}\left(z-z_j\right)\right)\right]\mu_j(z)=0, j=2,4. \tag{8}$$

The substitution of the independent variable transforms Eq. (8) into the Mathieu equation [27,28]

$$\frac{d^2\mu_j(t)}{dt^2}+\left(a-2q_j\cos(2t)\right)\mu_j(t)=0, \tag{9}$$

where $a=4\delta^2\left(\Omega-\langle\beta\rangle-h\right)/\alpha\pi^2$, $q_j=(-1)^{j/2^{-1}}\delta^2\Delta\beta/\alpha\pi^2$. Eq. (9) has solution

$$\mu_j(z)=\mu_j^+s\left(t,q_j\right)+\mu_j^-c\left(t,q_j\right), j=2,4, \tag{10}$$

where $c(t,q_j)$ and $s(t,q_j)$ are the general Mathieu functions.

In order to find the spectrum of SWs in the sample, we use the method of the transfer matrix [2,29]. If applied to individual atomic layers, as for example in Refs. [12,17,18,] this method can be used to calculate *numerically* SW spectrum of a MC with arbitrary interface profile, while here an analytical solution is obtained.

Let us introduce the following two-component column-vector

$$U(z)=\begin{pmatrix}\mu(z)\\ \sigma(z)\end{pmatrix},\ \sigma(z)=\frac{d\mu(z)}{dz}. \tag{11}$$

The values of the magnetization and its derivative in the beginning of each layer are expressed via their values in the end of the layer using the transfer matrix M_j

$$U\left(z_{j-1}\right)=M_jU\left(z_j\right). \tag{12}$$

For the basic layers ($j=1,3$), the transfer matrices are

$$M_{1,3}=\begin{pmatrix}\cos\left(k_{-,+}d\right) & -k_{-,+}\sin\left(k_{-,+}d\right)\\ k_{-,+}^{-1}\sin\left(k_{-,+}d\right) & \cos\left(k_{-,+}d\right)\end{pmatrix}, \tag{13}$$

where $k_{-,+}=\sqrt{\left(\Omega-h-\beta_{-,+}\right)/\alpha}$.

For the interface layers ($j=2,4$), the transfer matrices are

$$M_{2,4}=\begin{pmatrix}P_j & -\langle k\rangle Q_j\\ \langle k\rangle^{-1}P'_j & Q'_j\end{pmatrix},$$

where

$$P_j=\left[c'\left(0,q_j\right)s\left(t_j,q_j\right)-s'\left(0,q_j\right)c\left(t_j,q_j\right)\right],$$
$$Q_j=\left[c\left(0,q_j\right)s\left(t_j,q_j\right)-s\left(0,q_j\right)c\left(t_j,q_j\right)\right],$$
$$P'_j=\left[c'\left(0,q_j\right)s'\left(t_j,q_j\right)-s'\left(0,q_j\right)c'\left(t_j,q_j\right)\right],$$
$$Q'_j=\left[c\left(0,q_j\right)s'\left(t_j,q_j\right)-s\left(0,q_j\right)c'\left(t_j,q_j\right)\right],$$
$$c'\left(t_j,q_j\right)=\left.\frac{dc\left(t,q_j\right)}{dt}\right|_{t=t_j},\ s'\left(t_j,q_j\right)=\left.\frac{ds\left(t,q_j\right)}{dt}\right|_{t=t_j},$$
$$t_j=t(z_j),\ \langle k\rangle=\sqrt{\left(\Omega-h-\langle\beta\rangle\right)/\alpha}.$$

Due to the boundary conditions (5), vector $U(z)$ must be continuous at the layer boundaries. Consequently, values in the beginning and end of the period must be connected as

$$U(0)=MU(L),$$

where $M=\prod_{j=1}^{4}M_j$ is the transfer matrix for one period of the MC. Let us determine the eigenvalues λ of this matrix corresponding to eigenvectors $U(0)$.

$$\lambda U(0)=MU(0), \tag{14}$$

Eq. (14) coincides with the periodicity condition (6), if we assume that

$$\lambda=\exp\{iKL\}. \tag{15}$$

The eigenvalues λ are determined from the equation

$$\lambda^2+2\lambda\tilde{M}+1=0, \tag{16}$$

where

$$\tilde{M}=SpM,$$

$$\begin{aligned}\tilde{M}=&\left\{\begin{array}{c}\left[\cos(k_-d)P_2+(\kappa_-)^{-1}\sin(k_-d)P'_2\right]\cos(k_+d)+\\ +\left[\cos(k_-d)Q_2-(\kappa_-)^{-1}\sin(k_-d)Q'_2\right]\kappa_+\sin(k_+d)\end{array}\right\}P_4\\
&+\left\{\begin{array}{c}\left[\cos(k_-d)Q_2-(\kappa_-)^{-1}\sin(k_-d)Q'_2\right]\cos(k_+d)\\ -\left[\cos(k_-d)P_2+(\kappa_-)^{-1}\sin(k_-d)P'_2\right](\kappa_+)^{-1}\sin(k_+d)\end{array}\right\}P'_4\\
&-\left\{\begin{array}{c}\left[\ell k_-\sin(k_-d)P_2-\cos(k_-d)P'_2\right]\cos(k_+d)\\ +\left[\kappa_-\sin(k_-d)Q_2+\cos(k_-d)Q'_2\right]\kappa_+\sin(k_+d)\end{array}\right\}Q_4\\
&+\left\{\begin{array}{c}\left[\kappa_-\sin(k_-d)Q_2+\cos(k_-d)Q'_2\right]\cos(k_+d)\\ -\left[\kappa_-\sin(k_-d)P_2-\cos(k_-d)P'_2\right](\kappa_+)^{-1}\sin(k_+d)\end{array}\right\}Q'_4\end{aligned}$$

where $\kappa_\pm=\kappa_\pm/\langle k\rangle$.

Solving Eq. (16) and substituting the solution into Eq. (15), we find the SW spectrum of the MC

$$\cos(KL)=-\tilde{M}/2. \tag{17}$$

Fig. 2 shows the SW spectrum calculated from Eq. (17) for MCs of two different thicknesses. As expected, the spectrum has a band structure with magnonic band gaps at

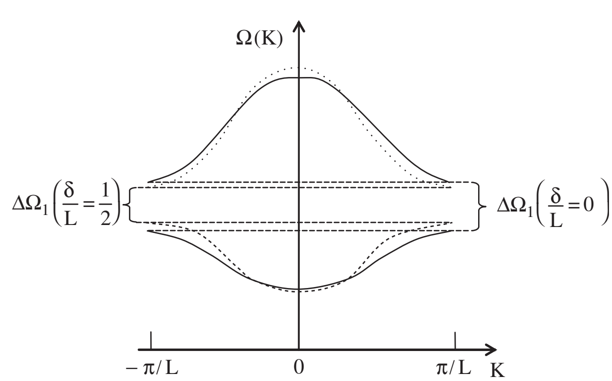

Fig. 2. The first two bands of the SW spectrum are shown for MCs with two different values of the interface thickness. ($\Delta\beta=2$, $\beta_-=2.0$, $h=0$).

the Brillouin zone boundaries corresponding to $KL = \pi \pm 2\pi n$. The width of the nth band gaps $\Omega_n(K)$ was determined numerically. In Fig. 3, the dependence of the width of the band gaps upon the depth of modulation of the anisotropy constant is presented for different (fixed) values of the interface thickness. The zero value of the interface thickness corresponds to the known Kronnig–Penney model [30] One can see that the band gap size for the different thicknesses of the interface is very similar to that in the Kronnig–Penney model at all depths of modulation of the anisotropy constant. Moreover, there are points in which the size of a particular band gap is identical in the models with smooth and sharp interfaces. Such points are also observed in Fig. 4 in which the ratio of the band gap width to the depth of modulation of the anisotropy constant is plotted as a function of the interface thickness. In Fig. 5, the depth of modulation of the anisotropy constant at which the width of the first band gap in the present model (interfaces of finite thickness) coincides with that in the Kronig–Penney model (infinitely thin interfaces) is shown for different interface thicknesses. It is interesting to note that this equality can only take place for a certain range of the depth of modulation of the anisotropy constant, and is not observed if the depth of modulation is either too small or too large.

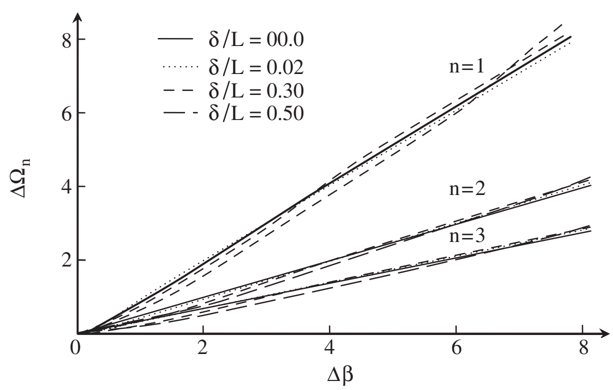

Fig. 3. The dependence of the width of the first three band gaps upon the depth of modulation of the anisotropy constant is shown for the different values of the interface thickness ($\beta_- = 2.0$, $h = 0$).

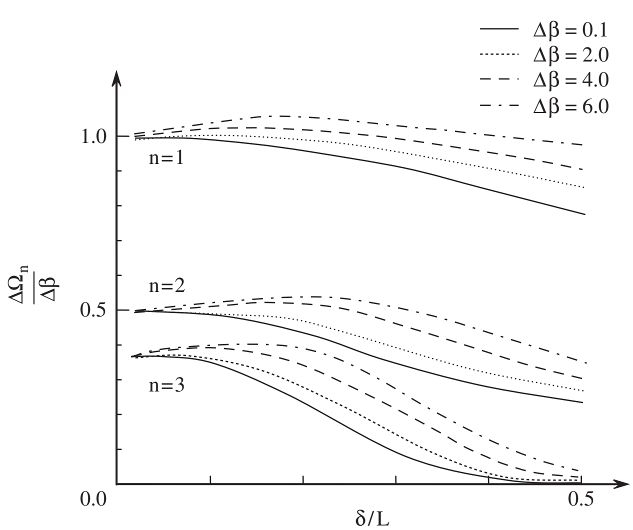

Fig. 4. The dependence of ratio of the width of the first three band gaps to the depth of modulation of the anisotropy constant upon the interface thickness is shown for the different values the depth of modulation of the anisotropy constant ($\beta_- = 2.0$, $h = 0$).

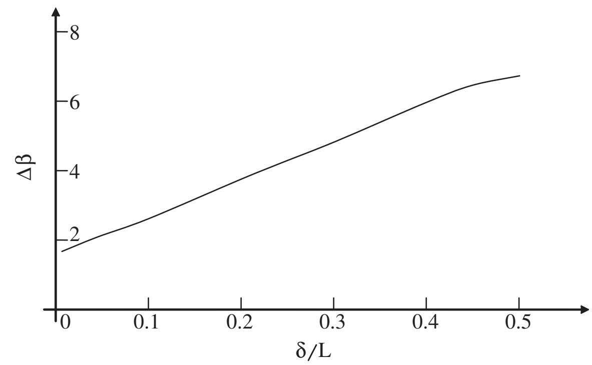

Fig. 5. The depth of modulation of the anisotropy constant at which the width of the first band gap in the present model (interfaces of finite thickness) coincides with that in the Kronig–Penney model (infinitely thin interfaces) is shown for different interface thicknesses.

In Refs. [21,22], Ignatchenko et al. showed that the size of the first band gap must increase when the interface thickness increases. In contrast, our Figs. 3 and 4 show that, depending upon the depth of modulation of the anisotropy constant, the band gap size can both decrease and increase when the interface thickness increases. We therefore conclude that, in order to determine the interface thickness from the spectral measurements of the size of the first and the third band gaps as it was proposed by Ignatchenko et al, one has to assume a particular coordinate dependence of the anisotropy constant at the interface, which may turn out to be quite arbitrary in practice.

In summary, the main conclusions of the present work are:

1. The smoothing of the magnetic interfaces often present in real MC modifies but does not destroy the magnonic band gap (stop band).
2. The size of the magnonic band gap depends not only upon the depth of modulation of the magnetic parameters and the thickness of the magnetic interface and but also upon the particular coordinate dependence of the magnetic parameters in the interface.

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