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Spin wave reflection from semi-infinite magnonic crystals with diffuse interfaces

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Abstract

The frequency dependence of the coefficient of spin wave reflection from a semi-infinite magnonic crystal with a periodically modulated value of the uniaxial anisotropy and a finite thickness of interfaces has been investigated, assuming a linear distribution of the anisotropy value in the interfaces. The analysis shows that the performance of magnonic devices employing magnonic crystals as a filtering element can degrade as the thickness of interfaces increases, e.g. due to the process of diffusion between constituent layers of the magnonic crystals.

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1. Introduction

Superlattices are artificial structures with a periodic modulation of one or several material parameters. Superlattices with periodically modulated magnetic parameters show magnonic band structure and are called magnonic crystals (MCs) when the spin wave (SW) wavelength of interest is comparable to their period of the MCs [1]. For SWs with wavelength much greater than the period of the MC, magnetic superlattices behave as magnetic meta-materials with properties different to those of the constituent layers [2]. Thus, the study of SWs in MCs (so-called magnonics [3]) is an intensively developing direction in the physics of magnetic phenomena and meta-materials.

The primary source of interest to magnonics is the opportunity to use SWs propagating in MCs as data carriers within elements of SW logic devices. In particular, it was suggested that the use of MC instead of continuous SW wave guides could help to decrease dimensions of such SW logic devices while maintaining their controllability by applied magnetic field [4]. In the context of electromagnetic meta-materials, MCs could offer a way of designing magnonic resonances so that regions of negative magnetic permeability and possibly of negative refractive index [5,6] could be created near the resonances, which could be tailored to reach frequencies of several THz [4].

As materials having spatial modulation of magnetic parameters, one can use yttrium–iron garnets grown

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using different chemical and physical methods, e.g. using liquid phase epitaxy with variable temperature regime [7], alloys of Fe_{100-x}–Ni_x and Co_{100-x}–P_x [8], and many other magnetic alloys with spatially varied concentration of the magnetic ions in layers, thus allowing for different values of magnetic parameters in layers with different x.

A great number of works has been dedicated to investigations of wave multilayered materials with infinitely thin interfaces [9–15]. However, in many cases, the assumption of infinitely thin interfaces is a severe idealization, since diffusion normally occurs between homogeneous layers of a multilayered structure, which can manifest itself, e.g. in formation of magnetically "dead layers" at the interface between magnetic and non-magnetic layers [16]. The diffusion is proportional to the concentration gradient at the interface and hence is fastest in multilayers prepared with sharp interfaces. On the other hand, multilayers could be prepared with smeared interfaces from the beginning, thus reducing the speed of subsequent inter-diffusion, provided of course that such multilayers retain their useful functionality, e.g. the magnonic band spectrum in the case of MCs. That is why models of MCs with finite thickness of interfaces have attracted an increasing interest of researchers recently.

For example, in Refs. [17–19] the influence of the interface thickness on the spectrum of spin waves (SW) in MC has been investigated, with a conclusion that the spectrum of waves depends significantly on the thickness of the interface. In Ref. [18], a possibility of recovering the magnetic structure of a multilayer material from its SW spectral characteristics has been demonstrated, assuming standing waves. However, measurements of scattering of propagating SWs from such samples might provide additional information regarding the MCs. So, in Ref. [20], the reflection coefficient of SWs from a MC with ideal interfaces was calculated. In Ref. [21], the authors investigated MCs with an imperfect exchange interaction between its adjacent layers. The main aim of the present work is to investigate the effect of the interface thickness on the coefficient of SW reflection from MCs.

2. Model of the material

Let us consider a semi-infinite MC represented by a system of two types of alternating homogeneous ferromagnetic layers of equal thicknesses. Each of the layer types is described by a different value of the uniaxial anisotropy. The direction of the easy axis of the anisotropy is assumed to be perpendicular to the plane

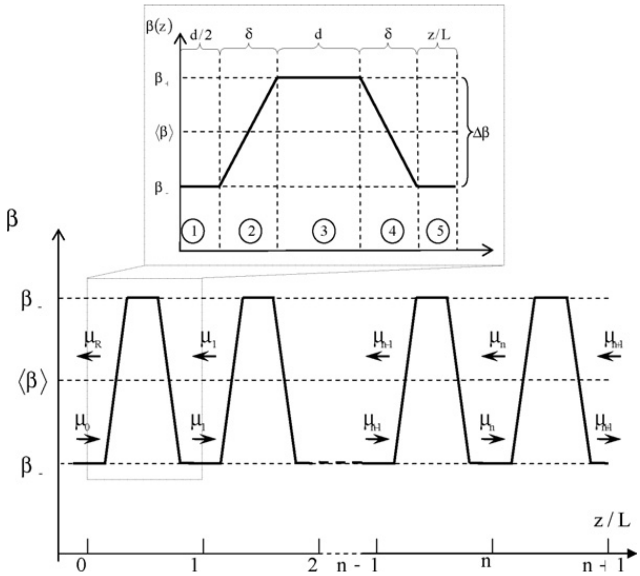

Fig. 1. The coordinate dependence of the anisotropy value β is schematically shown for a semi-infinite MC with interfaces of finite thickness and with a linear variation of the anisotropy value within the interfaces. The inset shows the same for a single period of the magnonic crystal.

of the layers. Also, it is assumed that the homogeneous "basic" layers of the MC are separated by inhomogeneous "transition" layers of finite thickness in which the value of the uniaxial anisotropy varies as

$$\beta(z) = \begin{cases} \beta_{1,5} = \langle\beta\rangle - \dfrac{\Delta\beta}{2} & z_{0,4} + nL < z < z_{1,5} + nL \\ \beta_3 = \langle\beta\rangle + \dfrac{\Delta\beta}{2} & z_2 + nL < z < z_3 + nL \\ \beta_{2,4} = \langle\beta\rangle \pm \dfrac{\Delta\beta}{2}\dfrac{z - z_{2,4}}{\delta} & z_{1,3} + nL < z < z_{2,3} + nL \end{cases} \quad (1)$$

where L is the period of the MC. Fig. 1 schematically shows the coordinate dependence of the anisotropy in the MC.

For the purpose of this calculation, the dynamics of the magnetization $\vec{M}(\vec{r}, t)$ can be described by the Landau–Lifshitz equation [22]:

$$\frac{\partial \vec{M}}{\partial t} = -g[\vec{M} \times \vec{H}_{\text{eff}}], \quad (2)$$

where $\vec{H}_{\text{eff}}$ is the effective magnetic field.

$$\vec{H}_{\text{eff}} = \left(H + \beta\left(\vec{M}\vec{n}\right)\right)\vec{n} + \vec{h}_m + \frac{\partial}{\partial \vec{r}}\left(\alpha \frac{\partial \vec{M}}{\partial \vec{r}}\right), \quad (3)$$

where α and g are the values of the parameter of the exchange interaction and the gyromagnetic ratio, respectively. H is the value of the external magnetic field, which is applied parallel to the easy axis direction, and $\vec{n}$ is the unit vector in the direction of the external magnetic field, $\vec{h}_m$ is demagnetizing field determined from the magnetostatic Maxwell equations

$$\mathrm{rot}\,(\vec{h}_m) = 0, \quad \mathrm{div}(\vec{h}_m) = -4\pi\,\mathrm{div}(\vec{M}). \qquad (4)$$

In a general case, $\vec{h}_m$ is coordinate-dependent and has a constant value only in a limited class of samples, namely in samples of ellipsoidal shape [23]. Here, we consider the case when the lateral dimensions of the sample are much greater than its total thickness in z direction and when the magnetization dynamics are uniform in the plane of the layers, i.e. the in-plane component of the SW wave vector is equal to zero. Furthermore, we assume that the saturation magnetization is constant throughout the sample $M_{j,0} = M_0$ where j is the number of a particular layer. In this case, the demagnetizing field is constant and leads to the so-called shape anisotropy equal to $-4\pi M_0$ and with axis perpendicular to the layers of the MC and hence parallel to the easy axis. Hence, for the sake of brevity, we assume that the shape anisotropy is accounted for in function $\beta(z)$, and therefore in equation (3) $\vec{h}_m = 0$.

If the absolute value of the magnetization was different in the two basic layers and varied continuously within the transition layers, $h_{m,z}$ would vary so that the normal component of the magnetic flux density $h_{m,z} - 4\pi M_z$ was constant. Again, this could be accounted for by assuming that function $\beta(z)$ also includes this contribution, making the same approximation that the variation of $h_{m,z}$ in the transition layers could be described by a linear function.

Let us consider small deviations $\vec{m}_j\,(j = 1, \ldots, 5)$ of the magnetization from the ground state, which is a uniform magnetization parallel to the easy axis,

$$\vec{M}_j(z, t) = \vec{n}M_0 + \vec{m}_j(z, t),\ |\vec{M}_j(\vec{r}, t)| = M_0,\ |\vec{m}_j| \ll M_0. \qquad (5)$$

Linearizing Eq. (2) with respect to Eq. (5), introducing the temporal Fourier components $\vec{m}_j\,(z, t) = \vec{m}_{j,\omega}\,(z)\,\exp\,\{i\omega t\}$ and then introducing variable $\mu = m_{x,\omega} + im_{y,\omega}$, we obtain the following equation that describes propagation of SWs in each layer of the MC

$$\frac{d^2\mu_j(z)}{dz^2} + k_j^2(z)\mu_j(z) = 0, \quad j = 1, \ldots, 5, \quad k_j(z) = \sqrt{\frac{\Omega - h - \beta_j(z)}{\alpha}}. \qquad (6)$$

where we introduced dimensionless frequency $\Omega = \omega/gM_0$ and magnetic field $h = H/M_0$. This allows us to avoid assuming any particular value of M_0, which for a wide range of magnetic materials of interest has values $M_0 \approx (10\ \text{to}\ 10^3)Gs$. In the following calculations, for the sake of brevity, we assumed that the period of the MC is $L = 5$ m km, the exchange parameter is $\alpha = 10^{-8}\,\mathrm{cm}^2$, and the value of the external magnetic field is $h = 0$. Yet, the formulae derived are more general and can be applied to a much wider range of samples.

At the interfaces, the solution of Eq. (6) must satisfy the boundary conditions of the continuity of the magnetization and its derivative [15].

In Ref. [24], Ignatovich developed an original recurrence method by which to derive the dispersion in and scattering of particles from a semi-infinite periodic potential. Here, we apply this method to the problem of SW scattering from a semi-infinite MC shown in Fig. 1.

Let us consider a spin wave incident on the boundary of the MC. The amplitudes of the incident wave μ_0 and of the reflected wave μ_r are connected as

$$\mu_r = R\mu_0,$$

where R is the coefficient of SW reflection from the MC.

Using the method proposed in Ref. [24], one can obtain

$$R = \frac{\sqrt{(\rho+1)^2 - \tau^2} - \sqrt{(\rho-1)^2 - \tau^2}}{\sqrt{(\rho+1)^2 - \tau^2} + \sqrt{(\rho-1)^2 - \tau^2}}. \qquad (7)$$

here ρ and τ are the reflection and transmission coefficients of SWs for a single period of the MC. Using the same method, it is also possible to obtain the Bloch wave vector in terms of the latter scattering coefficients

$$\exp\,(iKL) = \frac{\sqrt{(\tau+1)^2 - \rho^2} - \sqrt{(\tau-1)^2 - \rho^2}}{\sqrt{(\tau+1)^2 - \rho^2} + \sqrt{(\tau-1)^2 - \rho^2}}. \qquad (8)$$

Using the method of transfer matrices [25], one can obtain explicit expressions for scattering coefficients ρ and τ. According to this method, transfer matrix $\mathbf{M}_j$ is put in correspondence to each layer of the MC. This matrix connects the values of the magnetization and its derivative at the beginning and the end of that layer:

$$\left.\begin{pmatrix} \mu\,(z) \\ d\mu\,(z)/dz \end{pmatrix}\right|_{z_{j-1}} = \mathbf{M}_j \left.\begin{pmatrix} \mu\,(z) \\ d\mu\,(z)/dz \end{pmatrix}\right|_{z_j}. \qquad (9)$$

For the basic layers, these matrices are [26]:

$$\mathbf{M}_{1,5} = \begin{pmatrix} \cos\left(\dfrac{k_1 d}{2}\right) & -k_1 \sin\left(\dfrac{k_1 d}{2}\right) \\ k_1^{-1} \sin\left(\dfrac{k_1 d}{2}\right) & \cos\left(\dfrac{k_1 d}{2}\right) \end{pmatrix}, \qquad (10)$$

$$\mathbf{M}_3 = \begin{pmatrix} \cos\,(k_3 d) & -k_3 \sin\,(k_3 d) \\ k_3^{-1} \sin\,(k_3 d) & \cos\,(k_3 d) \end{pmatrix}$$

For the transition layers the transfer matrices are

$$\mathbf{M}_{2,4} = \begin{pmatrix} P_{2,4} & Q_{2,4}\ell \\ -P'_{2,4}\ell^{-1} & Q'_{2,4} \end{pmatrix}, \tag{11}$$

where for the linear profile of the interface considered here [26]:

$$P_j = \Gamma\left(\frac{1}{3}\right)3^{-1/3}\left[Ai(\zeta_j)3^{1/2} - Bi\left(\zeta_j\right)\right],$$

$$Q_j = \Gamma\left(\frac{2}{3}\right)3^{-1/2}\left[Ai(\zeta_j)3^{1/2} - Bi\left(\zeta_j\right)\right],$$

$$P'_j = \Gamma\left(\frac{1}{3}\right)3^{-1/3}\left[3^{1/2}\frac{dAi(\zeta_j)}{dz} - \frac{dBi(\zeta_j)}{dz}\right],$$

$$Q'_j = \Gamma\left(\frac{2}{3}\right)3^{-1/2}\left[3^{1/3}\frac{dAi(\zeta_j)}{dz} - \frac{dBi(\zeta_j)}{dz}\right],$$

$$\zeta_{2,4}(z) = -\frac{\ell^2}{\alpha}(\Omega - \beta_{\mp}) \pm \frac{\delta}{\ell},$$

$Ai(\zeta)$ and $Bi(\zeta)$ are the Airy functions, and $\ell = (\delta\alpha/\Delta\beta)^{1/3}$.

The transfer matrix for one period of the MC and its trace can be written as

$$\mathbf{M} = \prod_{j=1}^{5}\mathbf{M}_j, \quad \tilde{M} = \mathrm{Sp}\,\mathbf{M}, \tag{12}$$

Then the explicit expressions for the reflection and transmission coefficients will be

$$\rho = \sqrt{1 - 4\tilde{M}^2} \tag{13}$$

$$\tau = -2\tilde{M}, \tag{14}$$

where

$$\begin{aligned}\tilde{M} = \{&\left[\cos(k_1 d)P_2 + (\ell k_1)^{-1}\sin(k_1 d)P'_2\right]\cos(k_3 d) \\ &+\left[\cos(k_1 d)Q_2 - (\ell k_1)^{-1}\sin(k_1 d)Q'_2\right] \\ &\times \ell k_3\sin(k_3 d)\}P_4 + \Big\{\left[\ell k_1\sin(k_1 d)Q_2\right. \\ &\left.+\cos(k_1 d)Q'_2\right]\cos(k_3 d) - \left[\cos(k_1 d)P_2\right. \\ &\left.+(\ell k_1)^{-1}\sin(k_1 d)P'_2\right](\ell k_3)^{-1}\sin(k_3 d)\Big\}P'_4 \\ &-\Big\{\left[\ell k_1\sin(k_1 d)P_2 - \cos(k_1 d)P'_2\right] \\ &\times\cos(k_3 d) + \left[\ell k_1\sin(k_1 d)Q_2 + \cos(k_1 d)Q'_2\right] \\ &\times\ell k_3\sin(k_3 d)\Big\}Q_4 + \Big\{\left[\ell k_1\sin(k_1 d)Q_2\right. \\ &\left.+\cos(k_1 d)Q'_2\right]\cos(k_3 d) - \left[\ell k_1\sin(k_1 d)P_2\right. \\ &\left.-\cos(k_1 d)P'_2\right](\ell k_3)^{-1}\sin(k_3 d)\Big\}Q'_4.\end{aligned} \tag{15}$$

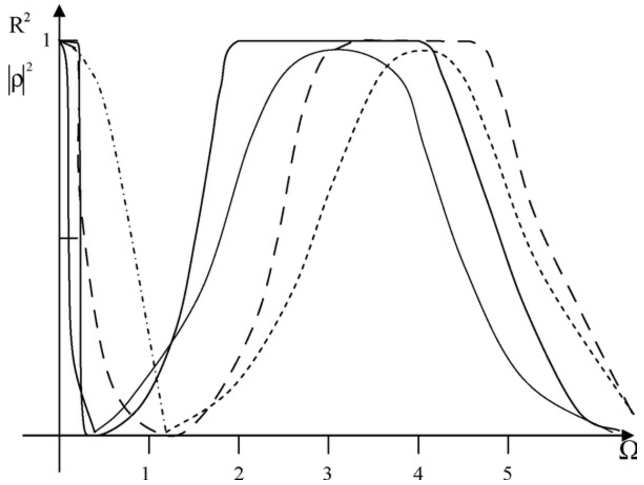

Fig. 2. The thick and thin lines show reflection coefficients $|R|^2(\Omega)$ and $|\rho|^2(\Omega)$, respectively for $\Delta\beta = 2$ and $\beta_- = 2.0$. The solid and dotted lines correspond to MCs with the thickness of interfaces equal to $\delta/L = 0.01$ and $\delta/L = 0.45$, respectively.

This allows us to investigate the intensity SW reflection coefficient $|R|^2(\Omega)$ in detail. The corresponding graph is shown in Fig. 2 for the earlier mentioned typical values of the magnetic parameters. For comparison, function $|\rho|^2(\Omega)$ is also shown in the same graph.

One can see that the intensity of the SW reflected from a semi-infinite MC with interfaces of finite thickness varies significantly as a function of the SW frequency, which is similar to the case of ideally sharp interfaces [20]. The frequency dependence reveals points of total transmission of the spin wave through the MC, as was also found in Ref. [21]. The presence of regions with $|R|^2 = 1$ indicates that at these frequencies the SW wave vector calculated from Eq. (8) is purely imaginary, which corresponds to band gaps in the magnonic spectrum of the MC. The origin of the band gaps can be traced back to maxima in the reflection from a single period of the MC. One can also see that the width of the band gaps and their positions depend on the thickness of the interface. At the same time, zeros of function $|R|^2(\Omega)$ again coincide with the zeros of $|\rho|^2(\Omega)$. The stacking of several periods of the MC leads to increasing the SW reflection coefficient and broadening of the regions of small and high reflection of SWs.

Fig. 3 shows the dependence of the frequency at which the total transmission of SWs through a MC is observed upon the thickness of interfaces. It is easy to see that the frequency varies sensitively with variation of the thickness of the transition layers. This demonstrates that the performance of magnonic devices containing MCs as a filtering element can vary significantly with time if the thickness of interfaces increases, e.g. due to the process of inter-diffusion between the constituent layers of the MCs [16].

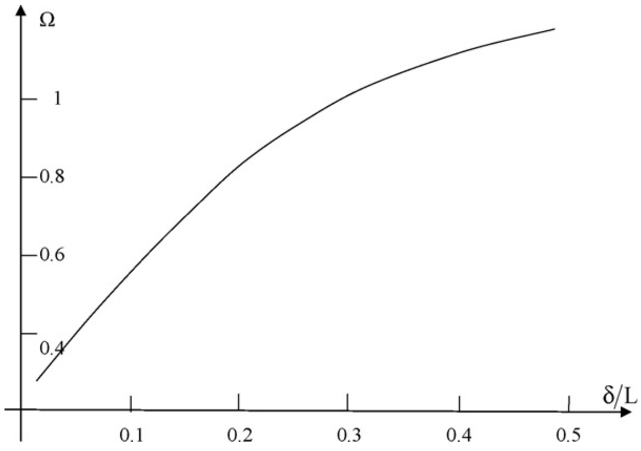

Fig. 3. Function $\Omega(\delta/L)$ for a total transmission of SWs through a MC is shown for $\Delta\beta = 2$ and $\beta_{-} = 2.0$.

3. Summary

We have used the transfer matrix method to investigate the coefficient of SW reflection from a semi-infinite MC with diffuse interfaces. The interfaces have been modeled by a linear distribution of the anisotropy value in transition layers between the main constituent layers of the MC. We have found that the coefficient of SW reflection from the boundary between the uniform ferromagnet and the semi-infinite MC varies non-monotonically with the SW frequency. In particular, regions of total reflection and points of total transmission have been observed. Although similar results have been previously obtained for ideal MCs, we find that the width and position of the regions of total reflection and the points of total transmission of spin waves depend sensitively upon the interface thickness. This fact has to be taken into account in the design of future magnonic devices.

References

[1] S.A. Nikitov, Ph. Tailhades, C.S. Tsai, J. Magn. Magn. Mater. 236 (2001) 320.
[2] V.V. Kruglyak, R.J. Hicken, A.N. Kuchko, V.Y. Gorobets, J. Appl. Phys. 98 (2005) 014304.
[3] V.V. Kruglyak, A.N. Kuchko, J. Magn. Magn. Mater. 272–276 (2004) 302.
[4] S.V. Vasiliev, V.V. Kruglyak, M.L. Sokolovskii, A.N. Kuchko, J. Appl. Phys. 101 (2007) 113919.
[5] A. Sihvola, Metamaterials 1 (2007) 2.
[6] E. Shamonina, L. Solymar, Metamaterials 1 (2007) 12.
[7] V.V. Tikhonov, A.V. Tolkachev, Fiz. Tverd. Tela (St. Petersburg) 36 (1994) 185.
[8] R.S. Iskhakov, A.S. Chekanov, L.A. Chekanova, Fiz. Tverd. Tela (St. Petersburg) 32 (1990) 441.
[9] R.P. van Stapele, F.J.A.M. Greidanus, J.W. Smits, J. Appl. Phys. 57 (1985) 1282.
[10] Y.I. Gorobets, A.E. Zyubanov, A.N. Kuchko, K.D. Shedzhuri, Fiz. Tverd. Tela (St. Petersburg) 34 (1992) 1486 [Phys. Solid State 34, 790 (1992)].
[11] K.Y. Guslienko, Fiz. Tverd. Tela (St. Petersburg) 37 (1995) 1603 [Phys. Solid State 37, 870 (1995)].
[12] M. Krawczyk, J.-C. Lévy, D. Mercier, H. Puszkarski, Phys. Lett. A 282 (2001) 186.
[13] A.M. Kosevich, JETP Lett. 74 (2001) 559.
[14] V.V. Kruglyak, A.N. Kuchko, Physica B 339 (2003) 130.
[15] V.V. Kruglyak, A.N. Kuchko, V.I. Finokhin, Fiz. Tverd. Tela (St. Petersburg) 46 (2004) 842 [Phys. Solid State 46, 867 (2004)].
[16] V.V. Kruglyak, A. Barman, R.J. Hicken, J.R. Childress, J.A. Katine, J. Appl. Phys. 97 (2005) 10A706.
[17] V.A. Ignatchenko, Y.I. Mankov, A.A. Maradudin, Phys. Rev. B 62 (2000) 2181.
[18] V.A. Ignatchenko, Y.I. Mankov, A.A. Maradudin, Phys. Rev. B 65 (2001) 024207.
[19] V.A. Ignatchenko, O.N. Laletin, Fiz. Tverd. Tela (St. Petersburg) 46 (2004) 2217 [Phys. Solid State 46, 2292 (2004)].
[20] Y.I. Gorobets, A.N. Kuchko, S.A. Reshetnyak, Fiz. Tverd. Tela (St. Petersburg) 38 (1996) 575 [Phys. Solid State 38, 315 (1996)].
[21] S.A. Reshetnyak, Low Temp. Phys. 33 (2007) 66.
[22] A.I. Akhiezer, V.G. Bar'yakhtar, S.V. Peletminskii, Spin Waves, Nauka, Moscow, 1967.
[23] V.G. Bar'yakhtar, V.N. Krivoruchko, D.A. Yablonsky, Green functions in magnetism theory, Naukova Dumka, Kyiv, 1984.
[24] V.K. Ignatovich, Usp. Fiz. Nauk. 150 (1986) 145 [Sov. Phys. Usp. 29, 879 (1986)].
[25] F.G. Bass, A.A. Bulgakov, A.P. Tetervov, High Frequency Properties of Semiconductor Superlattices, Nauka, Moscow, 1989.
[26] A.M. Kuchko, V.S. Tkachenko, Metallofiz. Noveishie Tekhnol. 27 (2005) 1157.