ISSN 0031-918X, The Physics of Metals and Metallography, 2006, Vol. 101, No. 6, pp. 513–518. © Pleiades Publishing, Inc., 2006.
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THEORY
OF METALS

Spectrum of Spin Waves in a Magnonic Crystal with a Structure Defect

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Received December 12, 2005

Abstract—Problem of the propagation of spin waves in a one-dimensional magnonic crystal with a structure defect has been analyzed theoretically. As a magnonic crystal, we consider a multilayer magnet representing an infinite system of plane-parallel alternate adjacent exchange-coupled slabs of magnets of two types. As a single structure defect that breaks the translational symmetry of the crystal, we consider a layer that is different from the layers composing the magnonic crystal. A dispersion law for spin-wave modes localized at the defect has been obtained. The appearance of several defect modes in the band gap has been analyzed depending on the parameters of the defect layer.

PACS numbers: 75.30.Ds

DOI: 10.1134/S0031918X06060019

INTRODUCTION

Physics of multilayer magnetic systems is one of the most intensely developing fields of the physics of magnetic phenomena. Technical progress in the field of obtaining films with thicknesses as small as a few nanometers and the possibility of obtaining structures with a predetermined distribution of components depending on the film thickness resulted in the creation of a new class of materials—artificial multilayer magnets, or magnonic crystals (MCs)—which are spin-wave analogs of photonic and phononic crystals [1–4] and represent materials with periodically modulated magnetic parameters. To date, numerous works devoted to the investigation of spin waves (SWs) and magnetostatic waves (MSWs) in MCs have been published. These works have shown, in particular, that in a system consisting of a very large number of periodic alternate layers and possessing a property of translational invariance there arises a band structure for SWs and MSWs instead of a discrete set of frequencies characteristic of the case of separate layers.

In real MCs, various structure defects (SDs) can exist, which lead to a local change in the magnitude of anisotropy and/or exchange interaction and in other parameters of the magnet and thereby violate the translational symmetry of the MC. Thus, it was shown in [5, 6] that a local change in the thickness of one of the layers composing an MC results in the appearance of a discrete level in the band gap. In more detail, the properties of localized modes in the photonic and phononic crystals in the presence of a single SD have been analyzed in [1, 3, 7–11]. The aim of this work was the derivation of a dispersion relation for a magnonic crystal with a structure defect and the investigation of the effect of its parameters on the number and properties of spin-wave modes localized at this defect.

MODEL OF A MAGNONIC CRYSTAL WITH A STRUCTURE DEFECT. FORMULATION OF THE PROBLEM

We consider the propagation of an SW in a one-dimensional MC with a single SD. As an MC, we take a multilayer magnet representing an infinite system of plane-parallel alternate adjacent exchange-coupled uniform and uniformly magnetized (to saturation) layers of two types with thicknesses a and b, constants of exchange coupling α_a and α_b, constants of uniaxial anisotropy β_a and β_b, gyromagnetic ratios g_a and g_b, and saturation magnetizations M_a and M_b. As a single SD breaking the translational symmetry of the MC, we take a layer, embedded into the multilayer material, of thickness d with magnetic parameters α_d, β_d, g_d, and M_d that are chosen in an arbitrary way, irrespective of the corresponding parameters of the ideal structure (Fig. 1). We assume that the easy axis (EA) of each layer lies perpendicular to the layers of the MC. A dc uniform applied magnetic field $\mathbf{H}$ is assumed to be oriented along the EA. The Cartesian coordinate system is chosen so that the axis OZ be perpendicular to the layer planes.

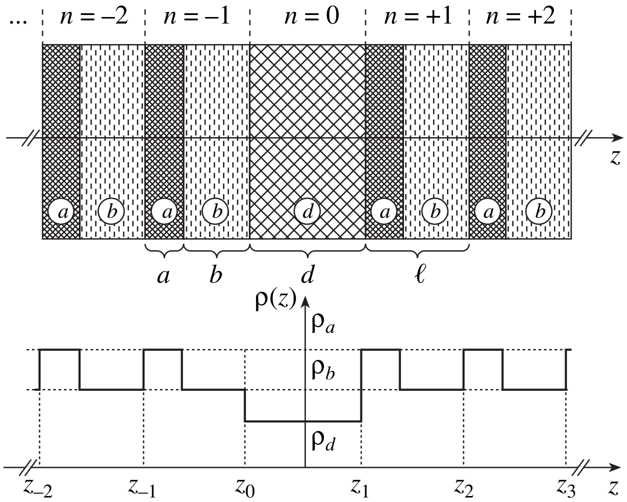

Fig. 1. Geometry of a one-dimensional magnonic crystal (MC) with a single structure defect (SD) and the coordinate dependence of the parameters of the material $\rho = \alpha, \beta, g, M$.

EQUATION OF MOTION OF MAGNETIZATION. BOUNDARY CONDITIONS

To describe the dynamics of the magnetization $\mathbf{M}(\mathbf{r}, t)$ of an MC, we use the Landau–Lifshitz equation [12]

$$\frac{\partial \mathbf{M}}{\partial t} = -g[\mathbf{M} \times \mathbf{H}_E], \quad (1)$$

where g is the gyromagnetic ratio ($g > 0$), and $\mathbf{H}_E$ is the effective magnetic field. At the chosen orientation of the magnetization in the direction perpendicular to the plane of layers, the surface magnetodipole fields reduce to the demagnetizing field. The corresponding magnetostatic components in this case can be included into the energy of magnetic anisotropy, which makes it possible to write the expression for the effective magnetic field in the form

$$\mathbf{H}_E = [H + \beta(\mathbf{M}\mathbf{n})]\mathbf{n} + \alpha\Delta\mathbf{M}, \quad (2)$$

where $\mathbf{n}$ is the unit vector along the OZ axis.

Consider small deviations $m(r, t)$ of the magnetization from the ground state—uniform magnetization along the EA. To this end, we write the magnetization as

$$\mathbf{M}(\mathbf{r}, t) = M_\nu \mathbf{n} + \mathbf{m}_\nu(\mathbf{r}, t), \text{ где } |\mathbf{m}_\nu| \ll M_\nu, \quad (3)$$

where the subscript $\nu = a, b, d$ denotes different types of layers.

For a monochromatic plane SW with a frequency ω and wave number k_ν which propagates in a direction perpendicular to the layer boundaries, we can write

$$\mathbf{m}_\nu(r, t) = \mathbf{m}_\nu \exp\{i\omega t\},$$
$$\mathbf{m}_\nu = A_\nu^+ \exp\{ik_\nu z\} + A_\nu^- \exp\{-ik_\nu z\}, \quad (4)$$

where $A_\nu^\pm$ are the amplitudes of SWs in the νth layer. In the approximation that is linear in $\mathbf{m}_\nu$, we obtain from Eq. (1) the following expression for the dispersion law of SWs in the homogeneous material of the νth layer:

$$k_\nu = \sqrt{\frac{1}{\alpha_\nu}\left(\frac{\omega}{g_\nu M_\nu} - \frac{H}{M_\nu} - \beta_\nu\right)}. \quad (5)$$

With allowance for (5), Eq. (1) takes on the form

$$\frac{d^2 m(z)}{dz^2} + \frac{1}{\alpha_\nu}\left(\frac{\omega}{g_\nu M_\nu} - \frac{H}{M_\nu} - \beta_\nu\right)m(z) = 0. \quad (6)$$

In the approximation of strong interlayer exchange coupling without spin pinning at interlayer boundaries, the solutions to Eq. (6) should satisfy boundary conditions of the following form [14, 15]:

$$\left.\frac{\mathbf{m}_a}{M_a}\right|_{z_n} = \left.\frac{\mathbf{m}_b}{M_b}\right|_{z_n}, \quad \left.\frac{J_a}{M_a}\frac{\partial \mathbf{m}_a}{\partial z}\right|_{z_n} = \left.\frac{J_b}{M_b}\frac{\partial \mathbf{m}_b}{\partial z}\right|_{z_n}, \quad (7)$$

where $J_\nu = \alpha_\nu M_\nu^2/2$.

SPECTRUM OF SPIN WAVES IN A MAGNONIC CRYSTAL

To find the spectrum of SWs in an MC, we use the method suggested in [8]. We introduce a two-component vector column $W(z) = (m(z), \chi(z))^t$. Using the transfer-matrix method [16], we can write the following expression that relates the magnitudes of the vector $W(z)$ at the beginning and at the end of each of the homogeneous layers (a, b, or d):

$$W(z_\nu^-) = S_\nu W(z_\nu^+),$$

$$S_\nu = \begin{pmatrix} \cos(k_\nu \nu) & \frac{1}{J_\nu k_\nu}\sin(k_\nu \nu) \\ -J_\nu k_\nu \sin(k_\nu \nu) & \cos(k_\nu \nu) \end{pmatrix},$$

where S_ν is the matrix of transformation of the homogeneous layer of type ν, and z_ν^- is the beginning, and $z_\nu^+ = z_\nu^- + \nu$ is the end coordinate of this layer.

By introducing a designation $W_n = X(z_n)$, where z_n is the coordinate of the beginning of the nth period (Fig. 1), the change undergone by W upon the propagation of the wave through one period of the MC can be written as

$$W_{n+1} = T_n W_n, \quad (8)$$

where T_n is the unimodular matrix of the transformation of the nth pariod ($n \neq 0$) or of the defect layer ($n = 0$) of the MC.

It is obvious that the transformation matrix for the defect layer is $T_0 = S_d$, and the transformation matrix for the nth period of the MC is $T_{n \neq 0} = S_b S_a$. Thus, for the components of the matrix $T_n = \begin{pmatrix} \lambda_n & \sigma_n \\ \zeta_n & \mu_n \end{pmatrix}$ we can write

$$\lambda_0 = \cos(k_d d),$$
$$\lambda_{n \neq 0} = \lambda = \cos(k_a a)\cos(k_b b) - \frac{J_a k_a}{J_b k_b}\sin(k_a a)\sin(k_b b);$$

$$\mu_0 = \cos(k_d d),$$
$$\mu_{n \neq 0} = \mu = \cos(k_a a)\cos(k_b b) - \frac{J_b k_b}{J_a k_a}\sin(k_a a)\sin(k_b b);$$

$$\sigma_0 = \frac{\sin(k_d d)}{J_d k_d},$$
$$\sigma_{n \neq 0} = \sigma = \frac{1}{J_a k_a}\sin(k_a a)\cos(k_b b) + \frac{1}{J_b k_b}\cos(k_a a)\sin(k_b b);$$

$$\zeta_0 = -J_d k_d \sin(k_d d);$$
$$\zeta_{n \neq 0} = \zeta = -J_a k_a \sin(k_a a)\cos(k_b b) - J_b k_b \cos(k_a a)\sin(k_b b).$$

With allowance for (4), the magnitude of the vector W at the boundary $z = z_n$ can be written as

$$W_n = PC_n, \tag{9}$$

where $C_n = (A_n^{(+)}, A_n^{(-)})^t$, $A_n^{(\pm)} = A_v^{(\pm)}\exp\{\pm ik_v z_n\}$, $P = M_v^{-1}\begin{pmatrix} 1 & 1 \\ iJ_v k_v & -iJ_v k_v \end{pmatrix}$.

Because of the continuity of W at the boundary $z = z_n$, the index v can be chosen arbitrarily. Equation (9) permits us to write Eq. (8) in the following form:

$$W_n = R_n C_{n+1}, \tag{10}$$

where $R_n = T_n^{-1} P$.

Combining Eqs. (9) and (10), we obtain the magnitudes of the magnetization at the boundary of the nth period ($m_n = m(z_n)$) in the form

$$m_n = A_n^{(+)} + A_n^{(-)} = (R_n)_{11} A_{n+1}^{(+)} + (R_n)_{12} A_{n+1}^{(-)}. \tag{11}$$

Following (8) and introducing a vector $V_n = (m_n, m_{n-1})^t$, the last relation can be reduced to the form

$$V_n = Y_{n-1} C_n, \tag{12}$$

where $Y_n = \begin{pmatrix} M_v^{-1} & M_v^{-1} \\ (R_n)_{11} & (R_n)_{12} \end{pmatrix}$.

Eliminating C_n from Eq. (12), we obtain, using Eqs. (9) and (10), that $T_n^{-1} P Y_n^{-1} V_{n+1} = P Y_{n-1}^{-1} V_n$. Passing in the last equation to the magnetizations in the layers, we obtain a discrete equation that relates the amplitude of the magnetization at the boundaries of adjacent layers in the form

$$\frac{1}{\sigma_n} m_{n+1} + \frac{1}{\sigma_{n-1}} m_{n-1} = \left(\frac{\lambda_n}{\sigma_n} + \frac{\mu_{n-1}}{\sigma_{n-1}}\right) m_n. \tag{13}$$

In the case of a defect-free MC, the transformation matrices T_n are the same for any period n. Therefore, the last equation is simplified and reduces to the form

$$m_{n+1} + m_{n-1} = F m_n, \tag{14}$$

where $F = \mathrm{Tr}(T_{n \neq 0}) = \lambda + \mu$.

Using the Bloch theorem and substituting $m_n \sim \exp\{in\kappa l\}$ (here, κ is the quasi-wave number, and $l = a + b$ is the period of MC) into Eq. (14), we obtain a well-known dispersion relation for the spectrum of SWs in a defect-free MC (see, e.g., [5, 16]:

$$2\cos(\kappa l) = F. \tag{15}$$

It is obvious that this equation will have real roots if the condition $|F| \leq 2$ is fulfilled and that it is this condition that determines the magnitudes of the allowed frequencies of SWs and the band character of the spectrum. The boundaries of the allowed bands can be found from the equation $|F| = 2$.

SPIN-WAVE MODES LOCALIZED AT THE DEFECT

Following [8], let us rewrite Eq. (13) in the form

$$(1 + \delta K_n) m_{n+1} + (1 + \delta K_{n-1}) m_{n-1} = (F + \delta Q_n + \delta N_{n-1}) m_n, \tag{16}$$

where the following designations are introduced: $\delta K_n = \sigma/\sigma_n - 1$, $\delta Q_n = \sigma(\lambda_n/\sigma_n - \lambda/\sigma)$, and $\delta N_n = \sigma(\mu_n/\sigma_n - \mu/\sigma)$. Taking into account that the quantities $\delta K_n = \delta K \delta_{n,0}$, $\delta Q_n = \delta Q \delta_{n,0}$, and $\delta N_n = \delta N \delta_{n,0}$, are nonzero only in the defect layer, we can write

$$\delta K = \frac{\sigma}{\sigma_0} - 1, \quad \delta Q = \sigma\left(\frac{\lambda_0}{\sigma_0} - \frac{\lambda}{\sigma}\right), \quad \delta N = \sigma\left(\frac{\mu_0}{\sigma_0} - \frac{\mu}{\sigma}\right).$$

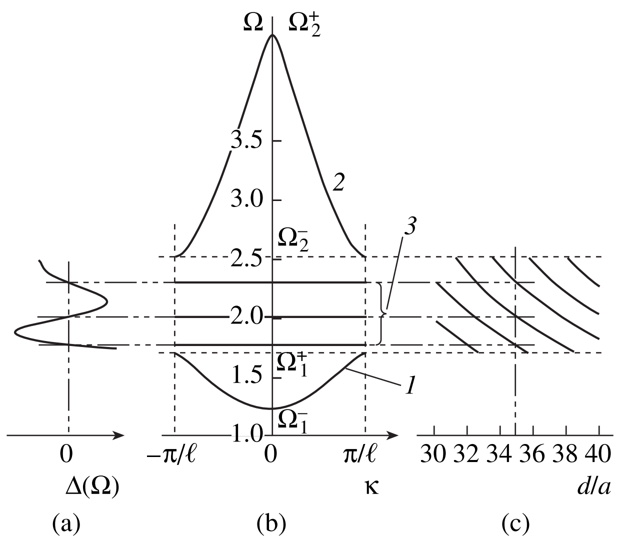

Fig. 2. Spectrum of spin waves (SWs) in an MC with an SD: $\alpha_a = \alpha_b = \alpha_d = \alpha$, $\beta_a = 1.8$, $\beta_b = 0.3$, $\beta_d = 0.6$, $a/\sqrt{\alpha} = 1$, $b/\sqrt{\alpha} = 2.5$, $d/\sqrt{\alpha} = 35$, and $H = 0$; horizontal dashed lines stand for the boundaries of the 1st band gap in the spectrum; Ω_n^- and Ω_n^+ are the lower and the upper boundaries of the nth allowed band; (a) the $\Delta(\Omega)$ dependence constructed based on Eq. (23); (b) the spectrum of SWs ((*1, 2*) 1st and 2nd allowed bands of the spectrum constructed based on Eq. (15); (*3*) discrete spin-wave modes localized at the SD); and (c) the dependence of the frequency of localized modes on the size of the SD (a fragment of Fig. 4 at d/a = 30–40).

Introducing the variables

$$L_{nk} = \delta_{n,k-1} + \delta_{n,k+1} - F\delta_{n,k},$$
$$\delta L_{nk} = (\delta K \delta_{n,k-1} - \delta Q \delta_{n,k})\delta_{n,0} + (\delta K \delta_{n,k+1} - \delta N \delta_{n,k})\delta_{n,1}, \quad (17)$$

we can rewrite Eqs. (16) in a simpler form

$$\sum_k (L_{nk} + \delta L_{nk}) m_k = 0, \quad (18)$$

and the set of equations (14) for a defect-free MC takes on the form

$$\sum_k L_{nk} m_k = 0. \quad (19)$$

Again following [8], we rewrite Eq. (19) in the Green's-function form as

$$\sum_l L_{nl} G_{lk} = \delta_{n,k} \quad (20)$$

or in an equivalent form

$$G_{n+1k} + G_{n-1k} - FG_{nk} = \delta_{n,k}.$$

Expanding G_{nk} into a Fourier series, we obtain for a defect-free MC the following expression for the Green's function:

$$G_{nk}(F) = \frac{1}{\pi}\int_0^{\pi} \frac{\cos[(n-k)\mu]}{2\cos\mu - F} d\mu = \gamma_{n-k}(F). \quad (21)$$

The last equation can be integrated analytically and yields the following expression for $\gamma_n(F)$:

$$\gamma_n(F) = -\frac{1}{\sqrt{F^2-4}}\left[\frac{F-\sqrt{F^2-4}}{2}\right]^{|n|}.$$

Using (21), we rewrite Eq. (18) with allowance for (20) in the form

$$\sum_k \left(\delta_{n,k} + \sum_m G_{nm}\delta L_{mk}\right) m_k = 0. \quad (22)$$

Since, in view of (17), the matrix δL has only four nonzero components, Eq. (22) reduces to the form

$$\begin{bmatrix} 1 - \gamma_0\delta Q + \gamma_1 \delta K & -\gamma_1 \delta N + \gamma_0 \delta K \\ -\gamma_1 \delta Q + \gamma_0 \delta K & 1 - \gamma_0 \delta N + \gamma_1 \delta K \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The condition of solvability of this system yields the following transcendental equation for determining discrete frequencies of localized modes [8]:

$$\Delta(\omega) = \gamma_0(\lambda_0 + \mu_0) - \gamma_1(\lambda\mu_0 + \lambda_0\mu - \sigma\zeta_0 - \sigma_0\zeta) = 0. \quad (23)$$

DISCUSSION OF RESULTS

Equation (23) takes into account the modulation of all parameters of the MC and SD. Because of the unwieldiness of the expression obtained, all subsequent analysis will be performed for the case of modulation of only the constant of uniaxial anisotropy and exchange ($g_a = g_b = g_d = g$; $M_a = M_b = M_d = M$). The graph of the $\Delta(\Omega)$ dependence ($\Omega = \omega/gM$) is given in Fig. 2a. The solutions to Eq. (23) (points of intersection of the graph with the axis $\Delta = 0$) were obtained numerically. These solutions determine the frequencies of spin-wave modes localized at the defect of the MC.

As is seen from Fig. 2b, the spectrum of SWs in the MC has a band character; its specific feature is the presence of forbidden gaps. The existence of band gaps makes impossible the propagation in the MC of those SWs whose frequency falls into a band gap. This feature of the spectrum is a consequence of translational symmetry inherent in the MC. As was noted in [5, 6], the presence of a defect in the MC violates the symmetry of the system, which makes possible the existence

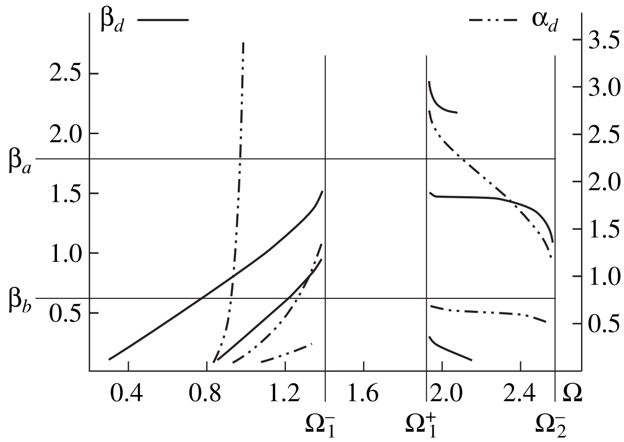

Fig. 3. Frequency of localized defect modes as a function of the magnitude of anisotropy (solid lines, left-hand axis) or exchange coupling (dot-and-dash lines, right-hand axis) of the structure defect (SD): $\alpha_a = \alpha_b = \alpha$, $\beta_a = 1.8$, $\beta_b = 0.6$, $a/\sqrt{\alpha} = 1$, $b/\sqrt{\alpha} = 2.5$, $d/\sqrt{\alpha} = 5$, and $H = 0$; solid lines, left-hand axis, $\alpha_d = \alpha$; dot-and-dash lines, right-hand axis, $\beta_d = 0.8$.

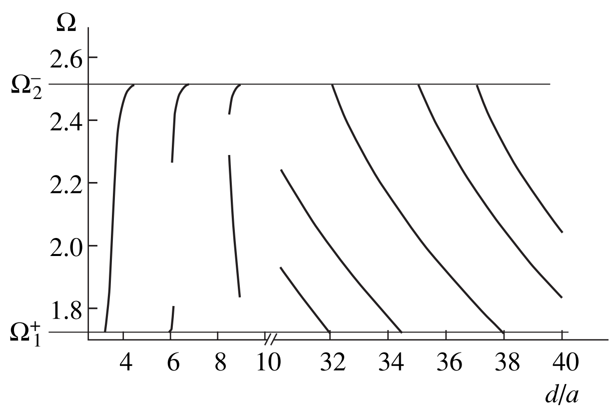

Fig. 4. Frequency of localized defect modes as a function of the size of the SD for the 1st band gap of the MC: $\alpha_a = \alpha_b = \alpha_d = \alpha$, $\beta_a = 1.8$, $\beta_b = 0.3$, $\beta_d = 0.6$, $a/\sqrt{\alpha} = 1$, $b/\sqrt{\alpha} = 2.5$, $H = 0$.

of modes with frequencies lying in the band gaps. These frequencies of SWs localized at defects are determined by solutions to Eq. (23). In [5, 6], the authors considered the disturbances of translational symmetry of MCs caused by changes in the length of one of the layers (a or b) in the SD. In this work, we consider an "extrinsic defect," i.e., a disturbance of the translational symmetry caused by the insertion of a homogeneous layer into the MC. An analysis of Eq. (23) shows that after the restoration of the translational invariance of the MC ($d = 0$), the discrete frequencies in the band gaps disappear.

At different values of the parameters of the MC and SD, Eq. (23) can have different numbers of roots, which results in a different number of localized modes. In particular, Fig. 3 displays the dependence of the frequency of localized defect modes on the magnitude of anisotropy and exchange of the SD. As is seen from the figure, the low-frequency SD ($\beta_d < \beta_a$, β_b) can lead to the appearance of localized modes in the zero band gap (with a frequency $\Omega < \Omega_1^-$ that is lower than the frequency of activation of SWs in the MC).

It is seen from Fig. 2 that in an MC with an SD, at certain values of the parameters of the MC and SD, there can exist additional (more than one) modes localized at the defect, apart from the localized mode whose existence was mentioned in [5, 6]. The dependence of the frequencies of localized modes on the SD thickness is displayed in Fig. 4.

An increase in the thickness of the SD at a constant magnitude of the anisotropy in the layer leads, as in the case of photonic crystals [3], to a growth of the number of localized modes. This result permits a simple qualitative interpretation [3]. An SD in an MC can be treated as a resonator (or a potential well), and the defect mode, as a standing wave arising in the resonator as a result of reflection from semibounded superlattices—walls of the resonator. As is known (see, e.g., [17]), the number of modes in a resonator (coupled states in a potential well) in a given frequency range is proportional to its length.

ACKNOWLEDGMENTS

We are grateful to Yu.I. Gorobets for a fruitful discussion of the results of this work.

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