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Spin wave spectrum of a magnonic crystal with an internally structured defect

A.N. Kuchko[a], M.L. Sokolovskii[a], V.V. Kruglyak[b,*]

[a]Donetsk National University, Universitetskaya 24, Donetsk 83055, Ukraine
[b]School of Physics, University of Exeter, Stocker road, Exeter EX4 4QL, UK

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Abstract

We report a theoretical investigation of the spin wave spectrum of a magnonic crystal with a defect. The latter is considered as either a single or double layer with parameters (the values of the uniaxial anisotropy and/or layer thickness) different from those of the constituent layers of the magnonic crystal. It is shown that, depending upon the parameters of the defect, the spectrum may contain either one or several additional discrete (localized) modes within the magnonic band gaps, or even a mini-band of those.

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The diversity of interesting phenomena and practical benefits discovered in the fields of photonics [1] and semiconductor superlattices [2] has led to a renewed attention to other materials possessing spatial periodicity. The band spectrum, which is the signature of the periodicity, is being intensively studied in plasmonic crystals [3], phononic crystals [4], periodic ferroelectric media [5], and carbon nanotubes in a transverse electric field [6]. The fact that the spectrum of spin waves (SW) in a periodic magnetic structure contains band gaps [7] has been known for some time [8,9]. However, the use of such structures for control of the SW propagation has been demonstrated relatively recently [10]. By analogy to photonic crystals and other periodic structures, periodic magnetic structures are referred to as *magnonic crystals* (MCs). Dielectric MCs are of additional interest because they also support magnetic field-controlled photonic band gaps [11]. Knowledge of their SW spectrum is therefore particularly important if they are to be operated (re-magnetized) at a high frequency.

*Corresponding author. Tel.: +44 0 1392 264163; fax: +44 0 1392 264111.
E-mail addresses: kuchko@dongu.donetsk.ua (A.N. Kuchko), V.V.Kruglyak@exeter.ac.uk (V.V. Kruglyak).

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In reality, the presence of structural imperfections can lead to a break down of the translational symmetry of a periodic medium. This has been shown to lead to the presence of localized modes in the spectra of photonic and phononic crystals [12–17]. For a MC, Nikitov et al. [18] showed that a local modification of the thickness of one of the layers leads to occurrence of one discrete level within the band gap. The aim of the present work is investigation of the SW spectrum for a MC with a defect of a more general form. We assume that the defect may be represented as either a single or double layer with parameters (the values of the uniaxial anisotropy and/or layer thickness) different from those of the constituent layers of the MC. We show that the spectrum may contain more than one defect modes in a particular band gap, and study the factors that determine the exact number of the defect modes.

Let us consider an infinite MC shown in Fig. 1 and consisting of periodically repeated thin film layers of two types, i.e. …ABABABAB…. The layers differ by their thicknesses (a and b) and values of the uniaxial anisotropy (β_a and β_b). The values of the spontaneous magnetization M_S and the exchange parameter α are assumed to be constant throughout the material. It is also assumed that the easy axis (EA) of the uniaxial anisotropy, the internal magnetic field $\mathbf{H}_{\text{int}}$ and the static magnetization direction are all perpendicular

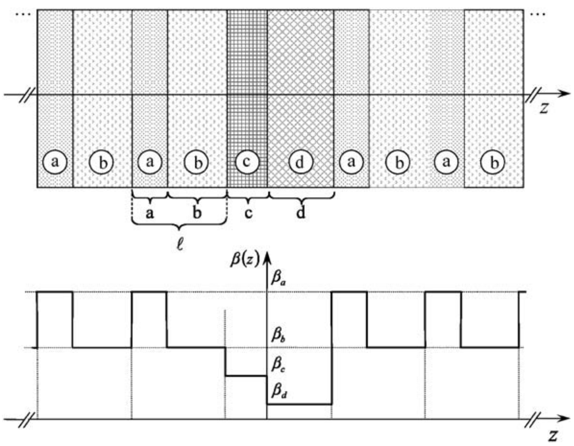

Fig. 1. The composition of the studied MC with a defect is schematically shown.

to MC's layers. We consider the internal rather than external static magnetic field so that, in addition to the latter, it also includes the static demagnetizing field due to a particular shape of the sample [19]. The defect is modeled as a double layer inserted into the MC. The thicknesses of the defect constituent layers are c and d, and their anisotropy values are β_c and β_d. This model also allows us to consider two different types of a single layer defect. When the thickness of one of the defect layers is equal to zero, the model describes a "defect of insertion", i.e. ABDAB. When the thickness and anisotropy of one of the defect layers are identical to those of the MC's normal layer that is not adjacent to this particular defect layer, the model describes a "defect of replacement", i.e. ABDBA. The difference between the two cases is that a MC with the defect of replacement possess a center of inversion symmetry, while a MC with the defect of insertion does not. The Cartesian coordinate system is defined so that its Z-axis is parallel to the EA.

The dynamics of magnetization $\mathbf{M}(\mathbf{r},t)$ is described by the Landau–Lifshitz equation [20]

$$\frac{\partial \mathbf{M}}{\partial t} = -g[\mathbf{M} \times \mathbf{H}_{\text{E}}], \quad (1)$$

where g is the gyromagnetic ratio ($g>0$) and $\mathbf{H}_{\text{E}}$ is the effective magnetic field. In the following, we will restrict ourselves to discussion of SW's that propagate parallel to the static magnetization. Then for the chosen geometry, dynamic magnetodipole fields do not contribute to the effective field [19], and we can write for the latter

$$\mathbf{H}_{\text{E}} = [H_{\text{int}} + \beta(\mathbf{Mn})]\mathbf{n} + \alpha\Delta\mathbf{M}, \quad (2)$$

where $\mathbf{n}$ is the unit vector parallel to the Z-axis, and for a thin film geometry $H_{\text{int}} = H - 4\pi M_S$, where H is the magnitude of the external static magnetic field.

Let us consider small deviations $\mathbf{m}(\mathbf{r},t)$ of the magnetization from the ground state, i.e. a uniform magnetization parallel to the EA. For this purpose, we represent magnetization as

$$\mathbf{M}(\mathbf{r},t) = M_S\mathbf{n} + \mathbf{m}(\mathbf{r},t), \quad \text{where} \quad |\mathbf{m}| \ll M_S. \quad (3)$$

Introducing notations $m_\pm = m_x \pm i m_y$ and seeking solutions in the form of harmonic waves

$m_{\pm}(\mathbf{r}, t) = m(z)\exp\{\pm i\omega\}$, we obtain the following linearized equation for $m(z)$

$$\frac{d^2 m(z)}{dz^2} + \left(\frac{\Omega - h - \beta(z)}{\alpha}\right) m(z) = 0, \tag{4}$$

where $\Omega = \omega/gM_S$ and $h = H/M_S$ are the dimensionless frequency and magnetic field. Within each of the uniform layers of the MC, Eq. (4) admits solutions

$$m(z) = A_v^{(+)} \exp\{+ik_v z\} + A_v^{(-)} \exp\{-ik_v z\}, \tag{5}$$

where index v denotes different layer types, being a, b, c, or d. $A_v^{(\pm)}$ are the SW amplitudes in the layers. The SW wave number within a particular layer is given by

$$k_v = \sqrt{\frac{\Omega - h - \beta_v}{\alpha}}. \tag{6}$$

At interfaces, solutions of Eq. (4) must satisfy the exchange boundary conditions [21,22] requiring the continuity of the magnetization $m(z)$ and its derivative $dm(z)/dz$.

As formulated above, the problem of finding the magnonic spectrum and the defect mode frequencies within the spectral band gaps is similar to that considered by Tamura in Ref. [14] for a phononic crystal of composition that is similar to ours. Following the "transfer matrix" method described in detail in Ref. [14], we obtain for the SW spectrum of a MC without defects

$$2\cos(\kappa l) = F = \lambda + \mu, \tag{7}$$

where κ is the Bloch wave number, l is the period of the MC, and

$$\lambda = \cos(k_a a)\cos(k_b b) - \left(\frac{k_a}{k_b}\right)\sin(k_a a)\sin(k_b b),$$
$$\mu = \cos(k_a a)\cos(k_b b) - \left(\frac{k_b}{k_a}\right)\sin(k_a a)\sin(k_b b). \tag{8}$$

The defect mode frequencies are given by solutions of equation

$$\Delta(\Omega) = \gamma_0(\lambda_0 + \mu_0) - \gamma_1(\lambda\mu_0 + \lambda_0\mu - \sigma\zeta_0 - \sigma_0\zeta) = 0, \tag{9}$$

where

$$\sigma = \frac{1}{k_a}\sin(k_a a)\cos(k_b b) + \frac{1}{k_b}\cos(k_a a)\sin(k_b b),$$
$$\zeta = -k_a\sin(k_a a)\cos(k_b b) - k_b\cos(k_a a)\sin(k_b b),$$
$$\lambda_0 = \cos(k_c c)\cos(k_d d) - \left(\frac{k_c}{k_d}\right)\sin(k_c c)\sin(k_d d),$$
$$\mu_0 = \cos(k_c c)\cos(k_d d) - \left(\frac{k_d}{k_c}\right)\sin(k_c c)\sin(k_d d),$$
$$\sigma_0 = \frac{1}{k_c}\sin(k_c c)\cos(k_d d) + \frac{1}{k_d}\cos(k_c c)\sin(k_d d),$$
$$\zeta_0 = -k_c\sin(k_c c)\cos(k_d d) - k_d\cos(k_c c)\sin(k_d d),$$
$$\gamma_0 = -\frac{1}{\sqrt{F^2 - 4}}, \quad \gamma_1 = \gamma_0\left[\frac{F - \sqrt{F^2 - 4}}{2}\right]. \tag{10}$$

Eq. (7) describes a spectrum with band gaps, which is shown in Fig. 2(a). The Brillouin zone boundaries are defined by condition $|F| = 2$. Within the band gaps, $|F| > 2$, and hence κ is

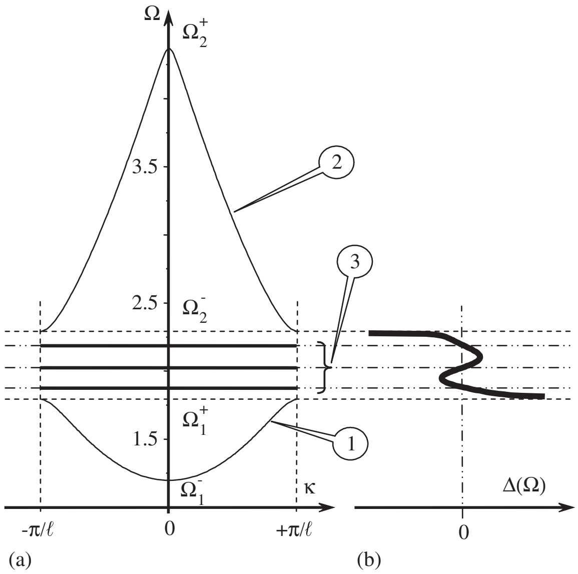

Fig. 2. The spectrum of the magnonic crystal with a defect is shown for $H = 0$, $\beta_a = 0.3$, $\beta_b = 1.8$, $\beta_c = 0.8$, $\beta_d = 1.1$, $a/\sqrt{\alpha} = 1$, $b/\sqrt{\alpha} = 2.5$, $c/\sqrt{\alpha} = 15$, $d/\sqrt{\alpha} = 25$. The horizontal dashed lines denote boundaries of the first band gap in the spectrum. $\Omega_n^{\pm}$ are the bottom/top boundaries of an nth band. In (a), 1 and 2 are the first and second bands of the spectrum plotted by Eq. (7), and 3 are the discrete spin wave modes localized on the defect. In (b), the frequency dependence of function $\Delta(\Omega)$ is plotted by Eq. (9).

imaginary. In this case, the values of the parameters defined in Eqs. (8) and (10) are all real. The frequency dependence of function $\Delta(\Omega)$ is shown in Fig. 2(b). The numerically obtained solutions of Eq. (9) define the frequency values of SW modes localized on the defect.

For different parameter values of the MC and the defect bilayer, Eq. (9) can have different number of roots, and hence localized modes. The dependence of the number and frequencies of the localized modes upon the anisotropy value and the thickness of the two types of a single layer defect is shown in Figs. 3 and 4, respectively. In particular, one can see from Fig. 3 that, when the anisotropy value of the defect layer is smaller than that in one of the MC's layers, defect modes emerge at frequencies lying below the lowest frequency of the first band of the MC spectrum. These modes are analogous to the localized electron states in a quantum well [23]. When the defect layer thickness is increased at a fixed anisotropy value, the number of the localized modes increases. This can also be interpreted at a qualitative level [12]. The defect layer in the MC acts as a resonator, and the defect modes are standing waves localized due to the reflection from its walls, which in this case have finite height. It is well known that, in a given frequency interval, the number of modes in a resonator is proportional to its length [23].

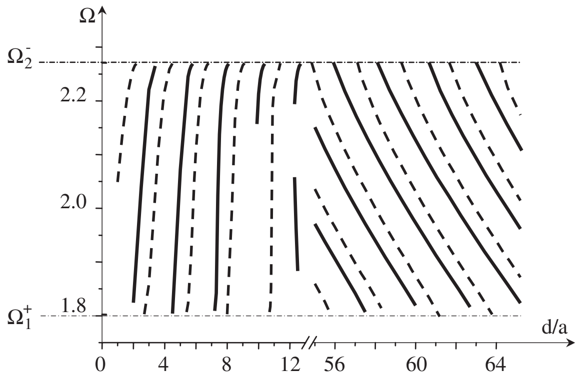

Fig. 4. The dependence of the defect mode frequencies in the first band gap upon the defect size is shown for $H=0$, $\beta_a=0.6$, $\beta_b=1.5$, $\beta_d=0.4$, $a/\sqrt{\alpha}=1$, $b/\sqrt{\alpha}=2.5$. The dashed and solid lines correspond to the defect of insertion ($c=0$) and the defect of replacement ($\beta_c=\beta_a$, $c=a$), respectively. The horizontal dash–dotted lines denote boundaries of the band gap.

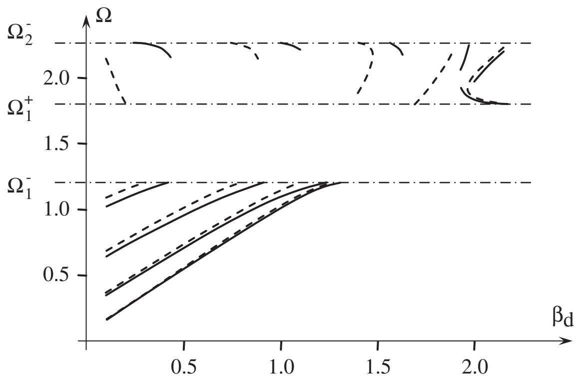

Fig. 3. The dependence of the localized mode frequencies upon the anisotropy value in the defect layer is shown for $H=0$, $\beta_a=0.6$, $\beta_b=1.5$, $a/\sqrt{\alpha}=1$, $b/\sqrt{\alpha}=2.5$, $d/\sqrt{\alpha}=10$. The dashed and solid lines correspond to the defect of insertion ($c=0$) and the defect of replacement ($c=a$, $\beta_c=\beta_a$), respectively. The horizontal dash–dotted lines denote boundaries of the band gaps.

In Fig. 4 for the layer thickness value of the defect of replacement about 12, the graph of the dependence of the localized mode frequency upon the defect layer size is almost vertical. There must therefore be a value of the defect layer thickness for which function $\Delta(\Omega)$ crosses zero only just. One may expect that, due to the finite damping [24,25] and scattering in the values of the parameters [26–28], which are always present in a real MC, the frequency at which function $\Delta(\Omega)$ intersects with zero will not be well defined, and so a mini-band of defect modes will be formed within the band gap.

In conclusion, we have shown that the presence of a single defect in a MC may lead, depending upon the defect size and composition, to appearance of either one or several defect modes, or a mini-band of those, within magnonic band gaps or below the smallest frequency allowed in a perfect MC.

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