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Spectrum of spin waves propagating in a periodic magnetic structure

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Abstract

In this paper, an analytical expression for the spectrum of propagating spin waves (SW) in a periodic multilayer ferromagnetic structure is obtained using Landau–Lifshitz (LL) equations. A graphical approach by which to study the influence of the modulation of many material parameters on character of SW propagation is proposed. New effects due to both the bias magnetic field and the joint modulation of several magnetic parameters of the structure are discovered. Namely, the parameters can be chosen so that the SW spectrum of the magnetic multilayer in a certain bias field will not contain band gaps. A slight misbalance of the system causes the appearance of the latter. The effect of dissipation on the results obtained as well as the possibility of their application is discussed.
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Recent advances in the field of spin wave (SW) experiments, including microwave SW resonance [1–3], Brilluoin light scattering [4,5], time- and space-resolved magneto-optical Kerr effect [6], and pulsed inductive microwave magnetometry [7,8], have allowed further insight into non-uniform spin dynamics in magnetic multilayers. Magnetic multilayers of periodic structure [1,2,4,9–16] (also referred to as magnetic superlattices (MS) or magnonic crystals) are particularly interesting

because of promise to create a new type of magnetoelectronic devices—those of *magnonics*, in which magnons would act as carriers of information [11]. Following the experimental demand, it is of importance now to study theoretically SWs in model systems that are as close as possible to those in reality. That is why a lot of effort has been devoted in recent years to theoretical investigations of SW phenomena in MS with different sorts of imperfections such as continuously distributed [14] and single [11] defects, disorder in arrangement [12] and finite thickness [10,13] of interfaces. However, so far only modulation of single parameters has usually been considered. Benefiting in simplicity and corresponding to some special cases of realizable

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in practice magnetic systems, this approach has allowed to reveal the basic common features of propagation and damping of SW in multilayer materials. But, for experiments and practical applications it is desirable to consider structures with modulation of all their parameters. This can not only make it feasible to apply obtained results to the diversity of materials used in practice but also, as we have recently shown [10,14], to discover new effects which are due to joint modulation of several parameters of a MS and are absent in simplified models with single parameter modulation.

In this work, the spectrum of propagating exchange SW in a periodic multilayer ferromagnetic structure is derived in case of modulation of all magnetic parameters included into Landau–Lifshitz (LL) equations without a relaxation term. The consequences of including the latter into consideration are qualitatively discussed. A more detailed discussion of SW damping in an MS will be given elsewhere [17]. Also, we propose a graphical approach for the study of joint influence of modulation of many parameters on the character of SW propagation and show how it helps to discover new interesting effects.

Let us consider a periodic structure consisting of alternating exchange coupled ferromagnetic layers of two types. Layers of both types are homogeneous and homogeneously magnetized up to saturation and characterized by thicknesses d_1 and d_2, parameters of inhomogeneous exchange interaction α_1 and α_2, constants of uniaxial anisotropy β_1 and β_2, values of the saturation magnetization M_1 and M_2, and gyromagnetic ratios g_1 and g_2 ($g_j > 0, j = 1, 2$), respectively. The in-plane easy axes of adjacent layers are parallel to each other as well as to the bias magnetic field $\vec{H}_0$. In order to be specific, we assume in the following that our MS is made of Co–P alloy [18] like those studied experimentally in Ref. [1]. Because of the unique sensitivity of their magnetic parameters to the P concentration, these alloys provide much freedom in designing properties of MS's. Particularly, systems can be made in which only chosen parameters are modulated while the others remain virtually constant [1,18]. The Cartesian coordinate system has been chosen so that its OX axis is

perpendicular to the layers and OZ axis is parallel to $\vec{H}_0$.

The magnetization dynamics is described by LL equations [19]

$$\frac{\partial \vec{M}}{\partial t}$$
$$= -g\left[\vec{M} \times \left\{ (H_0 + \beta(\vec{M}\vec{n}))\vec{n} + \vec{h}_m + \frac{\partial}{\partial \vec{r}}\left(\alpha \frac{\partial \vec{M}}{\partial \vec{r}} \right) \right\} \right], \tag{1}$$

where $\vec{M}$ is magnetization of the material, $\vec{n}$ is a unit vector along $\vec{H}_0$, and $\vec{h}_m$ is magnetic field derived by solving the magnetostatic Maxwell equations [9]

$$\mathrm{rot}(\vec{h}_m) = 0, \quad \mathrm{div}(\vec{h}_m + 4\pi\vec{M}) = 0.$$

Let us consider small deviations $\vec{m}$ of the system from its ground state-homogeneous (inside each layer) magnetization along $\vec{H}_0$:

$$\vec{M}(\vec{r}, t) = \vec{n} M_j + \vec{m}(\vec{r}, t),$$

where

$$|\vec{m}| \ll M_j.$$

If one considers a plane SW $\vec{m}(\vec{r}, t) = \vec{m}_{\omega, G_j, \vec{\kappa}} \exp\{i(\omega t \mp G_j x)\}$ with frequency ω, and perpendicular-to-plane wave vector $\vec{G}_j$ in linear on $\vec{m}$ approximation the spectrum of SW in the bulk of a j-type layer will be given by the well-known Kittel formula [20]

$$\omega^2 = g_j^2(H_0 + \beta_j M_j + \alpha_j M_j G_j^2)$$
$$\times (H_0 + \beta_j M_j + 4\pi M_j + \alpha_j M_j G_j^2). \tag{2}$$

In order to calculate the SW spectrum in the entire MS the Bloch theorem can be utilized [9,10]. This theorem states that because of the additional translation symmetry the solution of Eq. (1) can be represented as a modulated plane wave (so-called Bloch function). The modulating factors in it must have a period equal to that of the multilayer $d = d_1 + d_2$. Also, at the interfaces the exchange boundary conditions [21] have to be satisfied

$$\left.\frac{\vec{m}}{M}\right|_{x-0} = \left.\frac{\vec{m}}{M}\right|_{x+0}, \quad \left.\frac{A}{M}\frac{\partial \vec{m}}{\partial x}\right|_{x-0} = \left.\frac{A}{M}\frac{\partial \vec{m}}{\partial x}\right|_{x+0},$$

where

$$A = \tfrac{1}{2}\alpha M^2.$$

The first of these conditions reflects the fact that in an exchange dominated multilayer system without interlayer spacers the direction of the magnetization is a continuous function of coordinates. The second one ensures the conservation of the energy flow across an interface.

This brings us to the following form of the SW spectrum in the periodic multilayer magnetic structure with modulation of all its parameters:

$$\cos(G_1 d_1)\cos(G_2 d_2) - \frac{1}{2}\left(\frac{A_2 G_2}{A_1 G_1} + \frac{A_1 G_1}{A_2 G_2}\right)$$
$$\sin(G_1 d_1)\sin(G_2 d_2) = \cos(Kd), \qquad (3)$$

where K is a Bloch wave number.

Expressions (2) and (3) define implicitly K as a function of ω.

This sort of SW spectrum in a MS is general, and similar expressions have been derived before (for example, in the references cited above) in models with various assumptions about both properties of the layers and forms of interlayer coupling. They define a spectrum with band gaps–frequency domains in which propagation of waves is prohibited. Calculation of allocation and width of the band gaps from Eqs. (2) and (3) for a given set of the structure parameters is straightforward [10,14]. Different to this, an inverse problem of choice of the structure parameters in order to have band gaps in certain places of the SW spectrum is more complicated. In order to solve the problem, the following graphical method is suggested.

Let us take advantage of the fact that most of the information about the layer magnetic properties is hidden in G_j and introduce layer constants $q_j = G_j d_j$ and parameter p equal to the absolute value of lhs of Eq. (3) Then we plot diagrams (Fig. 1), where on the (q_1, q_2) plane band gaps corresponding to imaginary values of the Bloch wave number ($p > 1$) are shown with gray color. White color on the diagrams corresponds to allowed bands, where the Bloch wave number is real ($p < 1$). At the second stage lines of spectra are plotted on the same plane. The lines are given by parametrical dependence of the layer constants q_j upon SW frequency (2)

$$\omega(\alpha_1, \beta_1, g_1, M_1, d_1, q_1, H_0)$$
$$= \omega(\alpha_2, \beta_2, g_2, M_2, d_2, q_2, H_0). \qquad (4)$$

These diagrams allow one to determine boundaries of band gaps (crossings of the lines of spectra with the divides between black and white regions) for a given set of the parameters.

Let us first consider a structure where only the anisotropy and the gyromagnetic ratio are modulated, whereas the layer thickness, the saturation magnetization, and the exchange parameter stay constant throughout all the material. This case is shown in Fig. 1a. Here the bisector $q_1 = q_2$ (line 1) corresponds to a spectrum of homogeneous material. Regions next to this line are almost completely occupied with allowed bands. On the other hand, the size of the forbidden (black) regions is noticeably increased as approaching coordinate axes. One can see that by changing the depth of modulation of layer parameters or by varying magnitude of the bias magnetic field lines of spectra can be easily pushed in (lines 2 and 3) or out (lines 1 and 4) of the band gap rich regions. A similar situation takes place in the case of additional modulation of the exchange parameter, and (or) the saturation magnetization, although the map of the band gaps is different (Fig. 1b). It is interesting that in particular case $M_1 g_1 = M_2 g_2 = \langle M \rangle \langle g \rangle$ (note $M_1 \neq M_2, g_1 \neq g_2$) the bias field can be set to $H_c = -\langle M \rangle \langle \beta \rangle \Delta\beta/\Delta g (\Delta\varepsilon = (\varepsilon_2 - \varepsilon_1)/\langle \varepsilon \rangle, \langle \varepsilon \rangle = (\varepsilon_2 + \varepsilon_1)/2, \varepsilon$ – one of the MSs parameters) so that corresponding lines of spectra (4) degenerate into straight lines passing through the origin. Upon an additional condition $\alpha_1 M_1^4 = \alpha_2 M_2^4$, line $A_1 q_1/d_1 = A_2 q_2/d_2$ (line 3 in Fig. 1b) is obtained. It is interesting that this line represents, as can be also seen from Eq. (3), a SW spectrum of a periodic magnetic structure *without* band gaps. A deviation of the bias field magnitude from the value H_c frustrates the balance of the system and the band gaps emerge in the spectrum (line 4 in Fig. 1b).

The described peculiarities might be utilized in practice. Namely, such systems can be used as controlled by magnetic field functional elements of SW devices (switches or filters). We emphasize that these effects are absent in models with modulation of a single parameter but should exist in real structures where several parameters are usually modulated.

In this work all calculations are carried out in the dissipationless limit. The account of damping
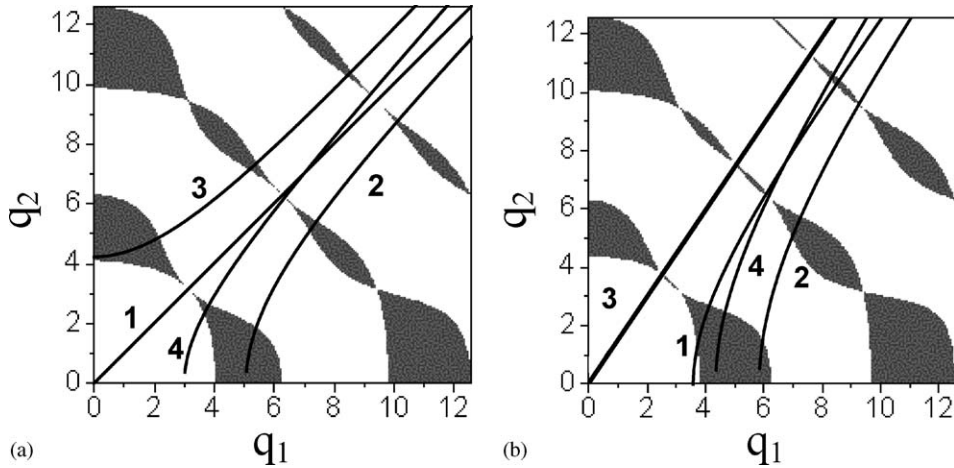
Fig. 1. Diagram for determination of the boundaries of band gaps in the spectrum of a periodic multilayer magnetic material is shown. The parameters of the material are defined as follows: $\langle M \rangle = 130$ emu, $\langle \alpha \rangle = 1.5 \cdot 10^{-12}$ cm^2, $\langle \beta \rangle = 23$, $\langle g \rangle = 1.76 \times 10^7$ s^{-1}Oe^{-1}, $\langle d \rangle = 0.2 \times 10^{-5}$ cm [18], $\Delta d = 0$, and (à) $\Delta \alpha = 0$, $\Delta M = 0$, (b), $\Delta \alpha \approx -0.76$, $\Delta M = 0.2$. The lines of spectra correspond to the following cases of modulation of the MS parameters. (1) $\Delta \beta = 0$, $\Delta g = 0$, $H = 0$; (2) $\Delta \beta = 0.4$, $\Delta g = 0$, $H = 6$ kOe; (3) $\Delta \beta = 0.4$, $\Delta g = -0.2$, $H = 6$ kOe; (4) $\Delta \beta = 0.4$, $\Delta g = -0.2$, $H = 0$. Note that lines a-1 and b-3 correspond to spectra *without* band gaps.

will cause the appearance of the imaginary part of the Bloch wave number in allowed bands of the spectrum [14]. Therefore, the diagrams will lose their meaning. Moreover, in the presence of spatial modulation of the parameter of magnetic viscosity the resulting SW damping in the system will depend in a non-trivial way upon the depth of modulation of the other parameters [14]. Nevertheless, in regions of band gaps of the dissipationless model, the damping of SW will remain big in comparison with allowed parts of the spectrum [14] and, hence, the conclusions of this work will be (at least qualitatively) valid.

To conclude, we have reported a theoretical investigation of the SW spectrum in a periodic ferromagnetic multilayer. Having taken into account periodic modulation of essentially all magnetic parameters describing such a system, we have shown how new effects emerge due to this and how these effects could be exploited in practice.

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