# Damping of spin waves in a real magnonic crystal

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## Abstract

An analytical expression for the effective damping coefficient (EDC) of propagating spin waves (SW's) in a magnonic crystal is obtained and analyzed. The analysis shows that the EDC depends strongly upon the SW frequency and the bias magnetic field. This dependence is more pronounced when damping is localized in vicinity of interfaces. It is shown that magnonic crystals with very big or very small EDC (for a given SW frequency range) can be designed by means of a proper choice of the depth of modulation of magnetic parameters.
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Spin waves (SWs) in multilayer magnets with artificial translation symmetry (also referred to as magnonic crystals (MC)) have recently attracted a great research interest ([1–6] and references therein). It is suggested that, similarly to photons in *photonics* and phonons in *phononics*, magnons might be used as carriers of information in systems studied in the emerging field of *magnonics*. The damping of SWs in a MC is particularly important for this field because only magnons with relaxation length bigger than characteristic dimensions of SW devices could be used. Also, for experiments it is desirable to consider structures with modulation of all magnetic parameters of MCs.

In this work, an analytical expression for the EDC of propagating exchange SWs is derived and analyzed for a ferromagnetic MC of general form. 1D periodical modulation of essentially all magnetic parameters of the system including constant of uniaxial anisotropy $\beta$, exchange parameter $\alpha$, saturation magnetization $M_0$, gyromagnetic ratio $g$, layer thickness $d$ as well as damping parameter of Landau type $\lambda$ is considered. The magnetization dynamics is described by Landau–Lifshits equation $\partial M/\partial t = -g[M \times H_E] - (\lambda/M^2)[M \times [M \times H_E]]$, where the effective magnetic field in short wave approximation is $H_E = (H_0 + \beta(Mn))n + (\partial/\partial r)(\alpha \partial M/\partial r)$. The magnetization is $M(r,t) = nM_0 + m(x,t)$, where $m$ is a small time dependent perturbation of the basic state, in which all layers are homogeneously magnetized up to saturation along unit vector $n$. The in-plane easy axes of layers as well as the external bias magnetic field $H_0$ are parallel to $n$, too. Adjacent layers are exchange coupled. Hence, exchange boundary conditions [2] $(m/M_0)_{x-0} = (m/M_0)_{x+0}$, $(\alpha M_0 \partial m/\partial x)_{x-0} = (\alpha M_0 \partial m/M_0 \partial x)_{x+0}$ must be satisfied. If $\chi$ is one of the listed above material parameters $\chi = \alpha, \beta, g, M_0, \lambda, d$ its the spatial distribution is given by

$$\chi(x) = \begin{cases} \chi_1, & x \subset [x_{2n} + 0.5d_{21}, x_{2n+1} - 0.5d_{12}], \\ \chi_2, & x \subset [x_{2n-1} + 0.5d_{12}, x_{2n} - 0.5d_{21}], \\ \chi_{12}, & x \subset [x_{2n+1} - 0.5d_{12}, x_{2n+1} + 0.5d_{12}], \\ \chi_{21}, & x \subset [x_{2n} - 0.5d_{21}, x_{2n} + 0.5d_{21}], \end{cases}$$

where constants $\chi_i$ and $\chi_{ij}$ ($i,j = 1,2$) refer to properties of main layers (except interfaces) and interfaces, respectively, $x_{2n} = nD$ and $x_{2n+1} = nD + d_1$ ($n = 0, \pm1, \pm2 \ldots$) are coordinates of the main layers boundaries, $d_1$ and $d_2$ are thicknesses of the main layers, $D = d_1 + d_2$ is the period of the MC. Interfaces are modeled as thin transition regions of thickness $d_{ij}$ that are centered at the main layers boundaries. The Cartesian coordinate

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system has been chosen so that $OX$-axis is perpendicular to the layers and $OZ$-axis is parallel to $\boldsymbol{H}_0$.

Because of the translation symmetry elemental excitations of a MC are described by their frequencies $\omega$ and Bloch wave numbers $K + iK$. Using the standard Bloch theorem technique [2], we derive in linear approximation on the thickness of interfacial layers for a SW propagating in a MC the following dispersion relation:

$$
\begin{aligned}
\cos((K + iK)D) = {} & \cos(G_1 d_1)\cos(G_2 d_2) \\
& - \xi_{1,2}\sin(G_1 d_1)\sin(G_2 d_2) \\
& - \cos(G_1 d_1)\sin(G_2 d_2)(\xi_{12,2} G_{12} d_{12} \\
& + \xi_{21,2} G_{21} d_{21} \\
& - 0.5(\xi_{1,2} G_1 + G_2)(d_{12} + d_{21})) \\
& - \cos(G_2 d_2)\sin(G_1 d_1)(\xi_{12,1} G_{12} d_{12} \\
& + \xi_{21,1} G_{21} d_{21} \\
& - 0.5(\xi_{1,2} G_2 + G_1)(d_{12} + d_{21})), \quad (1)
\end{aligned}
$$

$$
\xi_{j,k} = 0.5(A_k G_k/A_j G_j + A_j G_j/A_k G_k), \quad A_k = 0.5\alpha_k M_{0k}^2,
$$

$$
G_k = ((\omega/g_k M_{0k} - H_0/M_{0k} - \beta_k - i\omega\lambda_k/g_k^2 M_{0k}^2)/\alpha_k)^{1/2},
$$

$$
j, k = 1, 2, 12, 21.
$$

Expressions for the wave number $K$ and the EDC $\kappa$ can be written explicitly but are in general case rather complicated. For the sake of clarity, we consider here the most interesting case when only interfaces contribute noticeably to the SW damping. Then assuming $\lambda_1 = \lambda_2 = 0$, we obtain

$$
\kappa(\omega) = |C/D(1 - \cos^2(KD))^{1/2}|, \quad \text{where} \quad (2)
$$

$$
\begin{aligned}
C = {} & 0.5\omega(\lambda_{21} d_{21}/g_{21}^2 + \lambda_{12} d_{12}/g_{12}^2)(\cos(G_1 d_1)\sin(G_2 d_2)/ \\
& \alpha_2 M_{02}^2 G_2 + \cos(G_2 d_2)\sin(G_1 d_1)/\alpha_1 M_{01}^2 G_1)
\end{aligned}
$$

and $\cos(KD)$ is equal to rhs of (1) in which $G_k's$ are substituted by $\mathrm{Re}(G_k)'s$, defining spectrum $K(\omega)$.

Dependence of the EDC $\kappa$ upon the SW frequency and magnitude of the bias magnetic field is shown in Figs. 1 and 2, respectively. The flat zero-EDC regions are calculation artifacts and correspond to band gaps [2] where $|\cos(KD)| > 1$ and Eq. (2) is not valid. The main feature of the graphs is that because of the redistribution of the SW intensity within a MC the character of the $\kappa(\omega)$ and $\kappa(H_0)$ curves strongly depends on the depth of modulation of the material parameters. Particularly, a situation can be realized when for given frequency most of the intensity is concentrated in bulk of the layers (in the interfaces) and the EDC is reduced (enhanced) in comparison with the case of homogeneous material (Fig. 1). Because of the high localization of $\lambda$, this effect is more pronounced in the present model in comparison with the models considered in Ref. [2] where $\lambda$ was distributed inside layers uniformly. This finding gives a possibility of optimization of MC properties by a proper choice of depth of modulation of its parameters. Further control can be achieved by adjusting $H_0$ (Fig. 2).
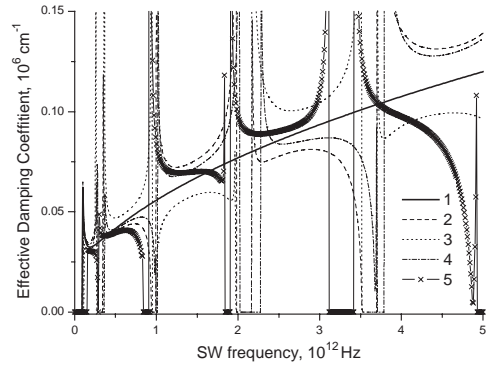


Fig. 1. Dependence of the EDC $\kappa$ upon the SW frequency $\omega$ as given by (2). The graphs correspond to the following cases of modulation of the parameters (depth of modulation $\Delta_\chi = (\chi_2 - \chi_1)/\chi_{\mathrm{av}}$, $\chi_{\mathrm{av}} = 0.5(\chi_2 + \chi_1)$): $\Delta d = 0$, $\Delta\beta = 0$, $H_0 = 0$, $\Delta\lambda = 0$, $\lambda_{21} = \lambda_{12}$, $\chi_{ij} = \chi_{\mathrm{av}}$, 1. homogeneous material; 2. $\Delta\alpha = 0.5$, $\Delta g = 0$, $\Delta M_0 = 0$; 3. $\Delta\alpha = 0$, $\Delta g = 0.5$, $\Delta M_0 = 0$; 4. $\Delta\alpha = 0$, $\Delta g = 0$, $\Delta M_0 = 0.5$; 5. $\Delta\alpha = 0.5$, $\Delta g = 0.5$, $\Delta M_0 = 0.5$.
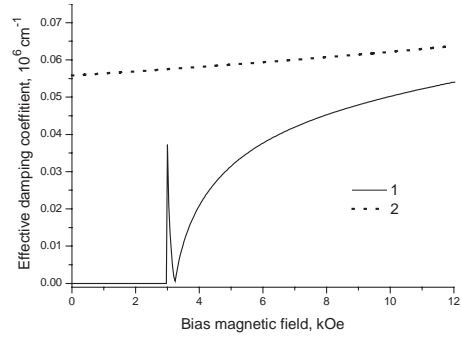


Fig. 2. Dependence of the EDC $\kappa$ upon the bias magnetic field $H_0$ as given by (2). The graphs correspond to the following cases of modulation of the parameters $\Delta d = 0$, $\Delta\beta = 0$, $\Delta\lambda = 0$, $\lambda_{21} = \lambda_{12}$, $\omega = 10^{12}$, $\chi_{ij} = \chi_{\mathrm{av}}$, 1. $\Delta\alpha = 0.5$, $\Delta g = 0.5$, $\Delta M_0 = 0.5$; 2. homogeneous material.

In conclusion, we calculated the EDC for a SW propagating in a MC of general type, and showed how a MC can be deliberately manufactured in order to reduce (or to enhance) SW damping in it.

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