

# **The Optical response of Single Interface and Thin Slab Dielectric Grating Structures**

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## **Abstract**

In this paper a modelling study of the optical response of sinusoidally corrugated dielectric grating structures is presented. Both single and dual interface structures are investigated for the case of TM polarised light normally incident upon gratings oriented in the classical mount. In the case of the dual interface structures the optical response of the structure as a function of the phase difference between the corrugations on the two interfaces is investigated. The amplitudes and phases of the diffracted orders produced from a single interface dielectric grating are obtained, and the results of this analysis are then used to explain the results obtained for the dual interface structures. There are three main results from the analysis of the dual interface structures: the suppression of the transmitted diffracted orders when the corrugations on the two interfaces are conformal, a blazing effect when there is a phase difference between the corrugations on the two interfaces, and the possibility of having almost all of the incident energy distributed between the  $\pm 1$  transmitted diffracted orders when the two corrugations are in anti-phase.

## 1. Introduction

Over recent years there has been a resurgence of interest in the optical response of structured metal surfaces while, to some measure, structured dielectrics have been overlooked. However, the optical response of textured dielectric surfaces has actually been a subject of interest for decades. Originally as a curiosity into the origin of diffraction, and more recently due to their many applications, large numbers of grating structures have been investigated, and many unexpected phenomena observed and explained. With the advent of increased computational power it has become possible to model the optical response of complex grating structures in order to more accurately design structures for specific applications, and to predict phenomena which may be subsequently investigated experimentally and utilized for new applications. This paper is such a model study and investigates the optical responses of both single and dual interface dielectric grating structures with, to the authors' knowledge, several previously unobserved phenomena being predicted for the first time.

In the first section of this paper, the optical properties of the simplest surface relief grating structure possible, that of the shallow dielectric grating in the classical mount with normally incident light, will be investigated. The analysis of non-blazed single interface dielectric structures appears to have been relatively ignored in the literature, presumably because the efficiencies of the diffracted orders are very small compared to those obtained from metallic gratings, therefore limiting their applications potential. The work which has been performed has tended to be based on developing computational methods to determine the efficiencies of the orders from such structures<sup>1-3</sup>.

The efficiencies and phases of the zeroth and  $\pm 1$  reflected and transmitted diffracted orders will be calculated for the case of TM polarised light normally incident from a lower index medium to a higher index medium, and from a higher index medium to a lower index medium. There are two main results from this analysis, which will be of use in the second

section of this paper. Firstly, the phase of the  $\pm 1$  diffracted order (either reflected or transmitted) with respect to the incident light may be either  $0^\circ$  or  $180^\circ$  (when the order is real and propagating) and this depends upon whether the light is incident from the high index medium or the low index medium. Also, when both reflected and transmitted diffracted orders are evanescent their phase is the same as when both orders are real and propagating, and in the case where only one of the reflected or transmitted diffracted orders is evanescent the phases of the diffracted orders vary depending upon the frequency of the incident light. Secondly, when a diffracted order in the semi-infinite medium which has a lower refractive index becomes evanescent, the intensity of the corresponding diffracted order in the higher refractive index medium (which is still a real propagating diffracted order) becomes zero. Correspondingly when the diffracted order in the higher index medium becomes evanescent the magnitude of the amplitude coefficient of the evanescent diffracted order in the lower refractive index medium becomes zero.

In the second section of the paper thin dielectric grating slabs, which are corrugated on both interfaces, are investigated. For dual interface structures, which are of major use in the communications industry, the majority of previous work has involved the behaviour of waveguide modes in the dielectric material of which the slab is made<sup>4-7</sup>. Rather than investigate the waveguide modes of the system, in this paper the effect of the corrugated slab structure on the diffracted orders is investigated, which, though they are affected by the waveguide modes, predominantly depend upon the interaction of the diffracted orders from the two interfaces. Even though the focus of this paper will be on the diffracted orders the effects of the corrugations of the two interfaces on the waveguide modes will be evident in the numerical results, although these will not be commented upon.

From the analysis on these slab structures it will be shown that, if the two surfaces are corrugated conformally, the transmitted diffracted order fields are reduced to almost zero. However, if the phase of the lower interface grating is changed relative to the grating on the

top interface this is no longer the case, and the system behaves as if it is effectively blazed. Previous work has only considered this blazing effect on the waveguide modes<sup>8,9</sup> and not on the diffraction efficiencies of the system. A further unexpected phenomenon occurs when the corrugations on the two surfaces are in anti-phase with each other, and under these conditions it is possible for almost all of the incident energy to be distributed equally between the  $\pm 1$  transmitted diffracted orders, with the intensity of the reflected and transmitted zeroth orders, and of the reflected diffracted orders being reduced to almost zero.

## 2. Single Interface Dielectric Gratings

The efficiencies and phases of the orders from a single interface dielectric structure can be calculated using various methods. In this paper the two methods which will be used are the Iterative Series Solution (ISS) method<sup>10,11</sup>, and a differential method based upon a coordinate transformation first developed by Chandezon<sup>12,13</sup>. Though the Chandezon method is more exact and flexible than is the ISS method (due to the fact that the ISS method is based upon the Rayleigh hypothesis and is only developed to calculate the optical response of single interface grating structures), the ISS method is useful since it allows the phase and magnitude of evanescent orders to be determined which will be of use later in this paper. A 400 nm pitch ( $\lambda_g$ ), 25 nm amplitude, sinusoidal grating will be studied, with the two dielectrics on either side of the grating interface described as air and SiO<sub>2</sub> (which is considered to be non-absorbing). The frequency dependent dielectric function of the SiO<sub>2</sub> is described by a polynomial fitted to experimentally determined values<sup>14</sup>:

$$n = 1.44561 + 1.02511 \times 10^{-18} \omega + 8.85533 \times 10^{-34} \omega^2, \quad k = 0$$

The efficiencies and phases of the zero and +1 orders for reflection and transmission for light incident from the air side of the interface are shown in figure 1, and for light incident

from the SiO<sub>2</sub> side in figure 2. (The results for the -1 orders are the same as for the +1 orders since only normally incident light is being considered).

It should be noted here that in both figures 1 and 2 the intensity of the orders is plotted for the case where the order is real and propagating, whereas the magnitude of the complex amplitude coefficient is plotted when it is evanescent. This means that in the case where the order is real and propagating the intensity falls to zero as the order becomes evanescent even though the magnitude of the complex amplitude coefficient does not. This is the cause of the suddenly discontinuous nature of the magnitude / intensity plots in these figures (for example at  $f \approx 0.5 \times 10^{15}$  Hz in figure 1(d)).

There are three main regions in figures 1 and 2: above  $f \approx 0.75 \times 10^{15}$  Hz the diffracted orders in both the air and the silica are real propagating orders, between  $f \approx 0.5 \times 10^{15}$  Hz the diffracted order in the SiO<sub>2</sub> is real and propagating, whereas in the air it is evanescent, and for below  $f \approx 0.5 \times 10^{15}$  Hz the diffracted orders in both the air and the SiO<sub>2</sub> are evanescent.

The approximate phases of the various orders for the case where all the orders are real and propagating (and, therefore, also when they are all evanescent) are shown in table 1.

	<b>Zeroth Order</b>		<b>+1 Diffracted Order</b>	
	<b>Air / SiO<sub>2</sub></b>	<b>SiO<sub>2</sub> / Air</b>	<b>Air / SiO<sub>2</sub></b>	<b>SiO<sub>2</sub> / Air</b>
<b>Reflection</b>	0	180	180	0
<b>Transmission</b>	0	0	0	180

**Table 1** The phases of the zeroth and +1 reflected and transmitted orders from both Air / SiO<sub>2</sub> and SiO<sub>2</sub> / Air single interface diffraction gratings

The origin of the differences in the phases can be seen by examination of the expressions used in the ISS method for the diffracted order amplitude coefficients. To first order these are,

$$e_{ip}^1(1) = \frac{(\varepsilon_2 - \varepsilon_1)ak_0^3\gamma_r\sqrt{\varepsilon_2}e_{ip}^0(0)}{2(\gamma_t + \gamma_r)(k_g^2 + \gamma_r\gamma_t)} \quad \text{Eqn 1}$$

for the +1 transmitted diffracted order, and

$$e_{ip}^1(1) = \frac{(\varepsilon_1 - \varepsilon_2)k_0ae_{ip}^0(0)}{4} \left[ \frac{(k_g^2 - \gamma_r\gamma_t)(\gamma_t - \gamma_r)}{(k_g^2 + \gamma_r\gamma_t)(\gamma_t + \gamma_r)} + 1 \right] \quad \text{Eqn 2}$$

for the +1 reflected diffracted order. In these equations  $a$  is the amplitude of the grating,  $\varepsilon_1$  is the dielectric function of the incident medium,  $\varepsilon_2$  is the dielectric function of the transmission medium,  $k_0$  is the wavevector of the incident light,  $k_g$  is the grating vector ( $= 2\pi / \lambda_g$ ),  $\gamma_r$  and  $\gamma_t$  are the wavevector components of the reflected and transmitted diffracted orders normal to the average plane of the grating, and  $e_{ip}^0(0)$  is the amplitude coefficient for transmission through a planar dielectric interface given by Fresnel's equations. The full iterative equations, as used in the numerical calculations contained within this paper, may be found in reference 11.

Though table 1 shows the phases of the orders for the cases where both the reflected and transmitted orders have the same character, in the region where one of the orders is evanescent, and one real and propagating, the phases do not remain nearly constant, but rather vary by approximately  $\pm 90^\circ$  as a function of the frequency throughout this region.

An interesting phenomenon occurs in the intensity / magnitude of the amplitude coefficient when the orders become evanescent. In the case of the air / SiO<sub>2</sub> interface, when the diffracted order in the air (the reflected diffracted order) becomes evanescent its amplitude coefficient does not become zero since evanescent fields still remain. However, the amplitude coefficient of the transmitted diffracted order in the SiO<sub>2</sub> (which is still real and propagating) reduces to zero at this point before increasing again as the frequency is lowered further. Similarly, when the transmitted diffracted order becomes evanescent the amplitude coefficient

of the reflected diffracted order reduces to zero before increasing once more as the frequency is lowered further.

In figure 3 a series of field profiles for the air / SiO<sub>2</sub> system for different frequencies is shown (obtained by using the numerical code based upon the method of Chandezon). The time averaged H<sub>z</sub> component of the fields (the z direction is into the page) are plotted since the z component of the H-field is the only component existing for TM polarised light with the grating in this orientation. Since the time averaged fields are plotted the propagating fields are averaged out. This leaves the beating between the incident and reflected zeroth order, and also the diffracted orders which are still observed since they form a beating with the propagating zeroth and incident / reflected orders.

Initially the reflected fields will be considered, starting with figure 3(f). At this frequency ( $f = 0.9 \times 10^{15}$  Hz) the reflected diffracted order is real and propagating, and the beating between the diffracted order and the incident and reflected zeroth orders is clear in the plot. When the frequency is reduced (figure 3(e),  $f = 0.75 \times 10^{15}$  Hz), the diffracted order becomes evanescent. However, the magnitude of the amplitude coefficient does not reduce to zero, even though the intensity of the diffracted order does, and for this reason the fields due to the diffracted order are still present (although they decay exponentially away from the interface this is not clear in this figure since the decay length is very long). Below this frequency (figures 3 (d), (c), (b), and (a),  $f = 0.7, 0.6, 0.514, 0.4 \times 10^{15}$  Hz) the reflected diffracted order remains evanescent and, as the frequency is reduced, the maximum magnitude of the evanescent diffracted order fields (at the surface) reduces (as would be expected from figure 1). It is also clear that the rate at which the evanescent order decays away from the surface increases. This is because the distance at which the evanescent orders have reduced to 1/e of their original value is proportional to  $1/\gamma_r$ , and as  $\gamma_r$  becomes larger (more imaginary) with reducing frequency, this distance reduces.

Similar behaviour is observed for the transmitted diffracted order, although in this case it is only the lowest frequency plot (figure 3 (a).  $f = 0.4 \times 10^{15}$  Hz) where the order is evanescent. However, in figure 3(e) the magnitude of the fields are much lower than for the other plots. This is the point at which the reflected diffracted order becomes evanescent and the reduction in these transmitted fields confirms the results of figure 1 which showed that the transmitted diffracted order amplitude coefficient reduces to zero at this point. Similar results have also been obtained for a  $\text{SiO}_2$  / air interface with the only difference being that the behaviour of the diffracted orders is reversed due to the reversed refractive indices.

### **3. Thin Corrugated Dielectric Slabs**

In this section the optical response of thin corrugated dielectric slabs will be considered. Initially the case of a conformally corrugated slab will be discussed where the corrugations on the two interfaces are identical and in phase with each other. Following this, the case of the asymmetric slab, where the corrugations on the two interfaces have some phase difference between them, will be considered. Throughout this section the optical response of the diffracted orders will generally be considered in terms of interference between the diffracted orders created at the top and bottom interfaces. However, it must be noted that this is only an approximation since it neglects the multiple reflections within the dielectric slab, but is useful for understanding the processes involved.

#### **3.1 Conformal structure**

The conformal system studied in this section is corrugated identically on both interfaces with a grating of pitch  $\lambda_g$  and amplitude  $a$ , has a thickness  $d$ , and phase difference between the corrugations of  $0^\circ$  (figure 4, but without the phase difference  $\Delta\phi$ ). In all of the remaining work in this paper the gratings will have a  $\lambda_g$  of 400 nm and an amplitude of 25 nm unless otherwise stated. The refractive indices of the media are such that all the media are



considered as non-absorbing with the refractive index of the slab being described as SiO<sub>2</sub> and that of the bounding media as air. Only TM polarised light normally incident upon the structure will be considered. The modelling throughout the rest of this paper has been obtained using the numerical modelling code based upon the method of Chandezon unless otherwise stated.

The intensities of the reflected and transmitted zeroth orders, as well as the reflected and transmitted +1 orders, as a function of the thickness of the dielectric slab and of the frequency of the incident light are shown in figure 5.

There are two different types of feature evident in figure 5. The sharp features (for example that which originates at  $f \approx 0.75 \times 10^{15}$  Hz for a slab thickness of zero) are the result of the waveguide modes in the system redistributing energy between the different propagating orders, and these will not be discussed any further in this paper. The broader modes which disperse to lower frequencies as the slab thickness is increased are a result of interference. For reflection, the light which is reflected from the first interface may interfere with the light reflected from the second interface, and this interference may be either constructive or destructive (depending upon the thickness of the slab). In reflection the condition for constructive interference to occur is that the light reflected from the bottom interface must be in phase with the light reflected from the top interface. Since (from table 1) the reflection from the top air / SiO<sub>2</sub> interface is 180° out of phase with respect to the incident light, and that reflected from the bottom interface is in phase with respect to the incident light, a phase change due to propagation of the light in the slab is required for the two reflected zeroth orders from the two interfaces to be in phase with each other and interfere constructively. This occurs first when the slab thickness is a quarter of the internal wavelength, since the light reflected from the bottom interface will then have to traverse half of the internal wavelength before it exits the system (in other words  $f = Nc/4nd$ , where  $f$  is the frequency of the incident light,  $N$  is an integer,  $c$  is the speed of light,  $d$  is the slab thickness, and  $n$  is the

refractive index. More generally we require  $f = \frac{c}{4nd}(2N + 1)$ . The same condition is required for destructive interference between the light which is directly transmitted through the structure, and the light which is reflected from the bottom interface, then reflected by the top interface, before propagating through the bottom interface. It is destructive interference in this case since there is a  $0^\circ$  phase change with respect to the incident light for transmission through both interfaces, and a  $180^\circ$  phase change for the light reflected from the bottom interface and then reflected from the top interface due to its additional propagation length of twice the slab thickness. Therefore, these two contributions to the transmitted zeroth order are out of phase with each other resulting in destructive interference.

These interference effects can also be observed in the reflected diffracted order plot, since the reflected diffracted orders created at the top interface may undergo interference with the reflected diffracted order created at the bottom interface in the same way as the reflected zeroth order (even though the phase changes upon diffraction for the two interfaces is the opposite to that of the zeroth order reflection the result is the same). However, these features will disperse differently with changing frequency, since they depend upon the incident angle of the diffracted order upon the bottom interface which will define the distance it traverses within the dielectric slab before exiting the system. Therefore, the slab thickness at which constructive interference may occur is different to that for the zeroth orders (particularly noticeable close to the critical edge), which are always propagating normal to the average plane of the surface.

The transmitted diffracted order is slightly different. In this case interference may occur between the transmitted diffracted order created at the top interface, and that created at the bottom interface. From table 1, the transmitted diffracted order created at the top interface is in phase with the incident light, whereas that created at the bottom interface is  $180^\circ$  out of phase with the incident light. Therefore, destructive interference will occur for a very thin slab

resulting in almost no intensity in the transmitted diffracted orders. The condition for constructive interference to occur is that the light diffracted at the top interface must travel half the wavelength of the internal light further than the light diffracted at the bottom interface. However, the only difference in the distance travelled by the two orders is due to the diffraction angle for the light diffracted at the top interface. Therefore, constructive interference can only occur when the diffraction angle at the top interface is very large, or when the frequency is very high (for a slab of thickness of the order investigated here). Of course, the phase difference between the light diffracted at the top interface and that diffracted at the bottom interface will change continuously as the slab thickness is increased and therefore there will be increasing diffraction from the system when this is the case. This can be clearly seen in figure 6, in which the time averaged fields for the system for incident light of frequency  $1 \times 10^{15}$  Hz for various slab thickness are plotted.

The analysis presented above is only for the region in which the diffracted orders are real and propagating. The other two possibilities; the case where diffraction in the SiO<sub>2</sub> slab is real and propagating, but the diffracted order in the air is evanescent, and the case where the diffracted order in both media are evanescent will now be considered. In figure 7 the field distributions for these two cases are shown for a slab of thickness 60 nm.

It is clear from figure 7(b) that when both diffractive orders are evanescent there are no transmitted diffracted fields, whereas when the transmitted diffracted order is real and propagating in the SiO<sub>2</sub> only (figure 7(a)) there is an evanescently decaying transmitted field. The same analysis method used previously (considering the two diffraction processes from the two interfaces individually and seeing how they interfere with each other) can be used to explain this. By using the ISS method used in the previous section and considering the two processes as: 1) the light is diffracted at the top interface and then transmitted through the bottom interface, and 2) the light is transmitted through the top interface and then diffracted by the bottom interface, the total transmitted diffracted order amplitude coefficient can be

obtained. For the 1<sup>st</sup> process the real and imaginary parts of the amplitude coefficient for the transmitted diffracted order created from the top interface, and also for light incident upon the bottom interface with  $k_x = k_g$  (since the light is normally incident this will be the wavevector of the +1 diffracted order for any frequency) are calculated. By combining these two results the total +1 diffracted field in the exit medium due to the two interfaces for light diffracted from the top interface can be obtained. The transmitted diffracted order fields due to diffraction from the bottom interface can also be calculated in a similar way, except that the zeroth order amplitude coefficient for light propagating through the top interface is calculated and combined with the amplitude coefficient for the diffracted order created at the bottom interface. These two results are then combined to give the total diffracted order fields for the two interface system, ignoring multiple reflections within the dielectric slab, and assuming that the slab thickness is zero since no account of the phase change due to propagation of the diffracted order within the slab thickness is incorporated within the model. The results of this are shown in figure 8.

It is clear from figure 8(c) that there are no transmitted diffracted order fields when the diffracted orders in the two media are real and propagating, and also when they are both evanescent. However, in the middle region, where the diffracted order is real and propagating in the SiO<sub>2</sub> only, evanescent diffraction in the exit medium of the system is evident. This result agrees with the results of the field distributions shown in figure 7. It is also evident that the lack of diffracted fields when the diffracted orders are both real and propagating, or both evanescent, is due to the fact that in these regions the phases of the two diffraction processes are approximately 180° out of phase with each other. This is not the case in the region where only the diffraction in the SiO<sub>2</sub> is real and propagating, and therefore transmitted diffracted order fields are observed.

## 3.2 Non-conformal structure

Having discussed the thin dielectric slab in a conformal geometry the case where the two gratings have identical modulations, but where the bottom corrugation is phase shifted with respect to the corrugation on the top interface, will now be considered. The system is shown in figure 4.

When the optical response of this structure is investigated both the +1 and -1 diffracted orders in reflection and transmission must be considered since the phase shift of the lower interface with respect to the top interface has broken the symmetry of the system. The optical response of a 60nm thick slab as a function of the phase difference between the corrugations on the top and bottom surfaces, and of the frequency of the incident light, is shown in figure 9. The grating parameters describing the two corrugations are the same as for the previously studied structures.

### 3.2.1 Antiphase grating

Initially the optical response when the two corrugations are in anti-phase will be considered. (When this is the case the structure is symmetric and the +1 and -1 diffracted orders will therefore be identical). The diffracted order intensities as a function of frequency for this case are shown in figure 10 (taken from figure 9)

It is clear that there is a very peculiar phenomenon occurring for this structure. For high frequencies almost all of the energy of the incident light is being transferred in to the  $\pm 1$  transmitted diffracted orders. The fact that the reflected zeroth and diffracted orders are very weak is no surprise since this will, in general, be the case from a dielectric grating structure. However, the reflected diffracted order is even smaller than would be expected when compared to those of the conformal case. Also, the reduction of the transmitted zeroth order to near zero intensity is somewhat unexpected.

Previously, the way in which the transmitted diffracted fields created from the top and bottom interfaces of a conformally modulated thin dielectric slab were out of phase with each other was discussed, and that due to this phase difference the total transmitted diffracted field from the structure was near zero (depending upon the slab thickness). When the two gratings are in anti-phase the diffracted orders created from the bottom surface are  $180^\circ$  out of phase as compared with those in a conformal geometry. This is because the position of the fields in the x-direction (parallel to the grating vector  $k_g$ ) of a diffracted order are 'locked' relative to the corrugation from which it originates such that the maxima and minima will always occur at the same position at the grating surface. In fact, this may be somewhat altered when there are two diffracting surfaces close to each other but is, in general, true. If the phase of the bottom corrugation is then altered with respect to the corrugation on the top interface this causes the phase of the diffracted orders created at the bottom interface *with respect to the incident light* to alter by the same amount as the phase difference between the corrugations. Thus, if the two corrugations are in anti-phase the diffracted orders created at the bottom interface will be  $180^\circ$  out of phase with those created at the bottom interface of the conformally corrugated structure.

For the reflected diffracted order in a conformal geometry, for the slab thickness considered here, the diffracted orders created from the two interfaces interfere constructively. For the anti-phase structure the opposite is true, and the diffracted orders created at the top and bottom interfaces cancel each other reducing the total reflected diffracted order fields from the system. The zeroth order reflected light is unaffected by the change in phase of the bottom surface corrugation but, since the zeroth order reflection is always going to be small (away from any possible waveguide modes of the system) the majority of the energy will be contained within the transmitted orders.

By the same arguments above, the transmitted diffracted order created at the bottom interface is also changed in phase by  $180^\circ$  when compared to that created at the bottom

surface for the conformally corrugated system. For the conformally modulated system the transmitted diffracted order fields from the top and bottom surfaces cancel, whereas for the anti-phase structure they interfere constructively.

It is interesting to consider the effect of the slab thickness, and of the grating amplitude, for an anti-phase grating in order to determine whether this property of almost all of the incident energy being transferred to the transmitted diffracted orders is a general property of anti-phase corrugated thin dielectric slabs, or whether it only happens to occur for the parameters investigated here. Therefore, the intensities of the various orders from the system as a function of frequency and slab thickness (figure 11), and also of the transmitted orders as a function of frequency and grating amplitude for a 150 nm thick dielectric slab (defined as the distance between the average planes of the gratings on the two interfaces) (figure 12) have been modelled.

Firstly, it is clear from figure 11 that the slab thickness has relatively little effect on the intensity of the transmitted diffracted order, except for a small periodicity as a function of slab thickness. This is caused by an increased propagation length within the dielectric slab. Therefore, this phenomenon seems to be relatively independent of the thickness of the grating slab.

However, the real key to the origin of the large transmitted diffracted order intensities is figure 12. The periodic nature of the diffraction efficiency into the  $\pm 1$  orders is clear from this figure, and it is well known that the diffraction efficiency from a single interface grating is also periodic. As stated previously the vast majority of the energy will be contained within the transmitted orders, and the two channels for each of the diffracted orders are in phase with each other due to the  $180^\circ$  phase difference between the corrugations on either side of the structure, whilst the two channels resulting in the transmitted zeroth order (from direct propagation through the interfaces and from diffracted orders from the first interface being diffracted back into the zeroth order at the second interface) will be out of phase. All that is

required for the effect described here is for the diffraction efficiency to be such that the two channels contributing to the zeroth transmitted order cancel to near zero, which will automatically result in a maximum in the  $\pm 1$  diffracted orders. Since the reflected zeroth order is bound to be small, and the reflected  $\pm 1$  orders are also small because they are the result of the cancellation of two out of phase diffraction processes, the  $\pm 1$  transmitted diffracted orders will each contain approximately 50% of the incident energy.

### 3.2.2 Arbitrary phase grating

The case where the phase of the bottom surface corrugation is out of phase with the top surface corrugation by some angle  $\Delta\phi$  will now be considered. From figure 9 it is clear that, for a particular frequency and  $\Delta\phi$ , the intensity of the +1 and -1 reflected diffracted orders are different, even though the light is normally incident upon the structure. Therefore, the structure exhibits the same type of behaviour as a blazed grating. However, the transmitted diffracted order intensities for the +1 and -1 orders are almost the same, which would not be expected from a blazed grating structure. This is shown in figure 13.

Using the simple ray model for the diffraction properties of these two interface systems with no multiple reflections within the dielectric slab this phenomenon may be understood. The contributing phase changes of the transmitted and reflected diffracted orders created at the top and bottom surfaces are shown in table 2.

	<b>Diffraction From</b>	<b>+1</b>	<b>-1</b>
<b>Reflected</b>	Top Surface	$\phi^t$	$\phi^t$
	Bottom Surface	$\phi_{+1}^b + \Delta\phi^d + \Delta\phi_{+1}^u$	$\phi_{-1}^b + \Delta\phi^d + \Delta\phi_{-1}^u$
<b>Transmitted</b>	Top Surface	$\phi^t + \Delta\phi_{+1}^d$	$\phi^t + \Delta\phi_{-1}^d$



	Bottom Surface	$\phi_{+1}^b + \Delta\phi^d$	$\phi_{-1}^b + \Delta\phi^d$
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**Table 2:** The phase effects which need to be considered when determining the total  $\pm 1$  reflected and transmitted diffracted order fields from a thin corrugated dielectric slab (ignoring multiple reflections).

In table 2 the following notation has been used. If there is any difference in the phase between the +1 and  $-1$  diffracted orders then the diffracted order has been added as a subscript. The t superscript denotes diffraction from the top surface, the b superscript denotes diffraction from the bottom surface, and the u and d superscripts denote any phase change due to propagation in the dielectric slab in the upward and downward going directions.

Firstly, the transmitted diffracted orders will be considered. The only terms which are different for the +1 and  $-1$  transmitted diffracted orders are the  $\phi_{\pm 1}^b$  and  $\phi_{\pm 1}^d$  terms. The  $\phi_{+1}^b$  and  $\phi_{-1}^b$  terms are in fact  $\pm\Delta\phi$  (the phase difference between the corrugations on the two interfaces). Since these must be symmetric about  $0^\circ$ , and for this situation  $\phi^t = 0^\circ$  (table 1) they are also symmetric about  $\phi^t$ . Therefore, when the top and bottom diffraction processes are combined to give the total transmitted diffracted order, the  $\phi_{\pm 1}^b$  terms will not cause any difference between the  $\pm 1$  orders. Similarly, the  $\phi_{\pm 1}^d$  terms are approximately symmetric about  $\phi^d$  and will also have no effect upon the total transmitted diffracted field. Therefore, since both  $\phi_{\pm 1}^b$  and  $\phi_{\pm 1}^d$  are symmetric about terms occurring in the other diffraction process it would be expected that the +1 and  $-1$  transmitted diffracted order intensities would be very similar even for a non-symmetric grating system.

Unlike the transmission case, in reflection the terms occurring in the bottom surface corrugation diffraction process which are different for the  $\pm 1$  diffracted orders ( $\phi_{\pm 1}^b$  and  $\phi_{\pm 1}^u$ ) are not symmetric about terms occurring in the top surface corrugation diffraction process. In

this case the  $\phi_{\pm 1}^b$  terms are symmetric about  $\phi^i$  (as in the transmission case) but the  $\phi_{\pm 1}^u$  terms are not, and this results in a different phase difference between the top and bottom surface diffraction processes for the +1 and -1 reflected diffracted orders, and thus a blazing effect results. Also, since this difference is a result of the propagation of the light in the dielectric slab for the bottom surface diffraction process, it would be expected that it would be frequency dependent. This agrees with the results given in figure 9.

#### **4. Conclusions**

In this paper an investigation into the optical response of single and dual interface sinusoidally corrugated dielectric structures has been presented. In the first section the zeroth and  $\pm 1$  diffracted order intensities / magnitudes and phases for the diffracted orders has been described. It was found that when either the reflected or transmitted diffracted order becomes evanescent the amplitude coefficient of the other diffracted order reduces to zero even though the amplitude coefficient of the diffracted order which has become evanescent still has some magnitude. It has also been shown that the phase of the diffracted orders depend upon whether the light is incident from the air side of the structure, or the SiO<sub>2</sub> side of the structure.

In the second section the results obtained on single interface structures have been used to explain results obtained from thin dielectric slabs corrugated on both interfaces. There were three main results from this investigation: 1) That there is virtually no transmitted diffracted order from a conformally corrugated structure, 2) Almost all of the incident energy may be equally distributed between the  $\pm 1$  transmitted diffracted orders when the corrugations on the two interfaces are in anti-phase, and 3) An effective blazing effect occurs in the reflected diffracted orders when there is a phase difference between the two corrugations which is neither 0° or 180°.

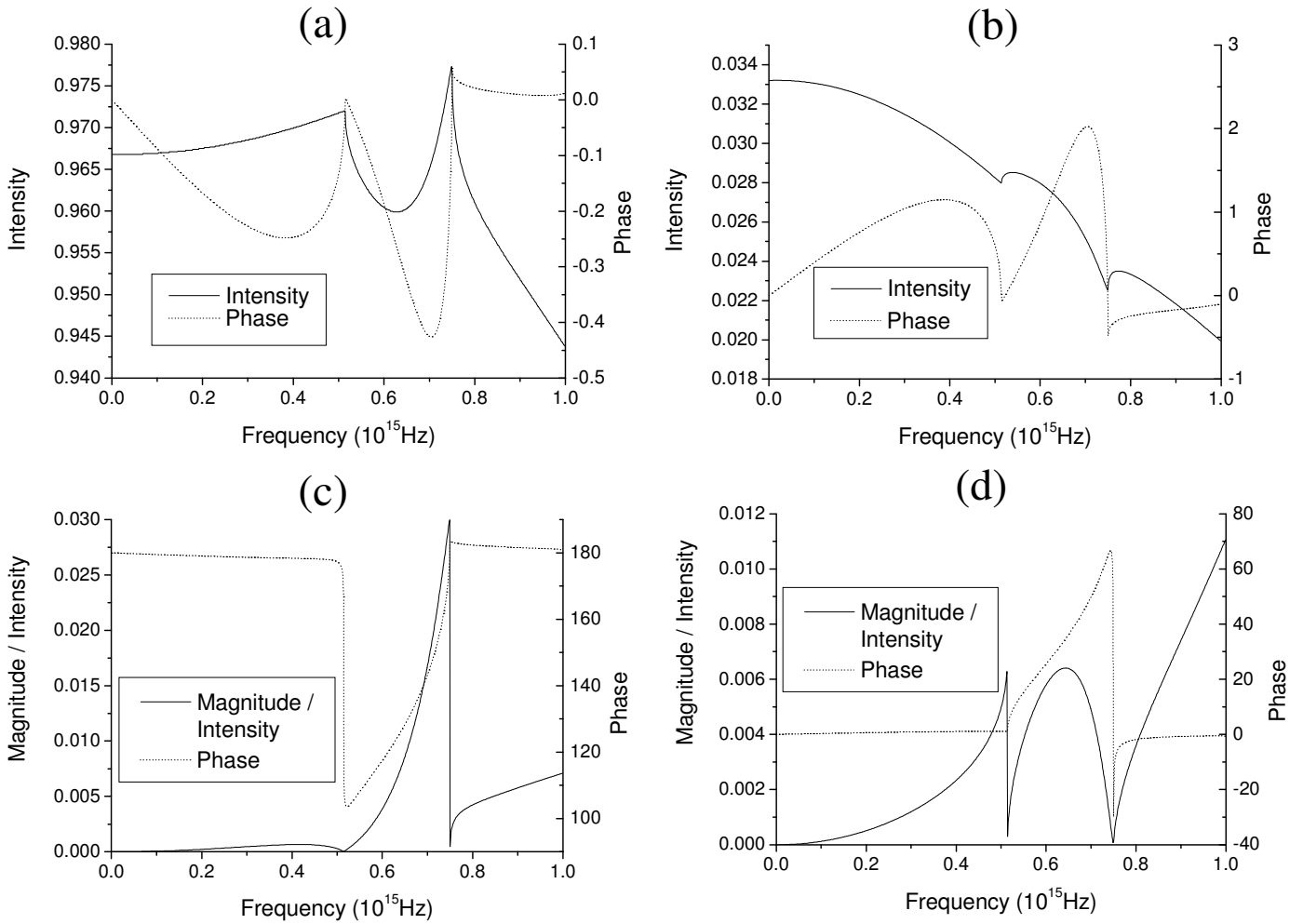
## **Acknowledgements**

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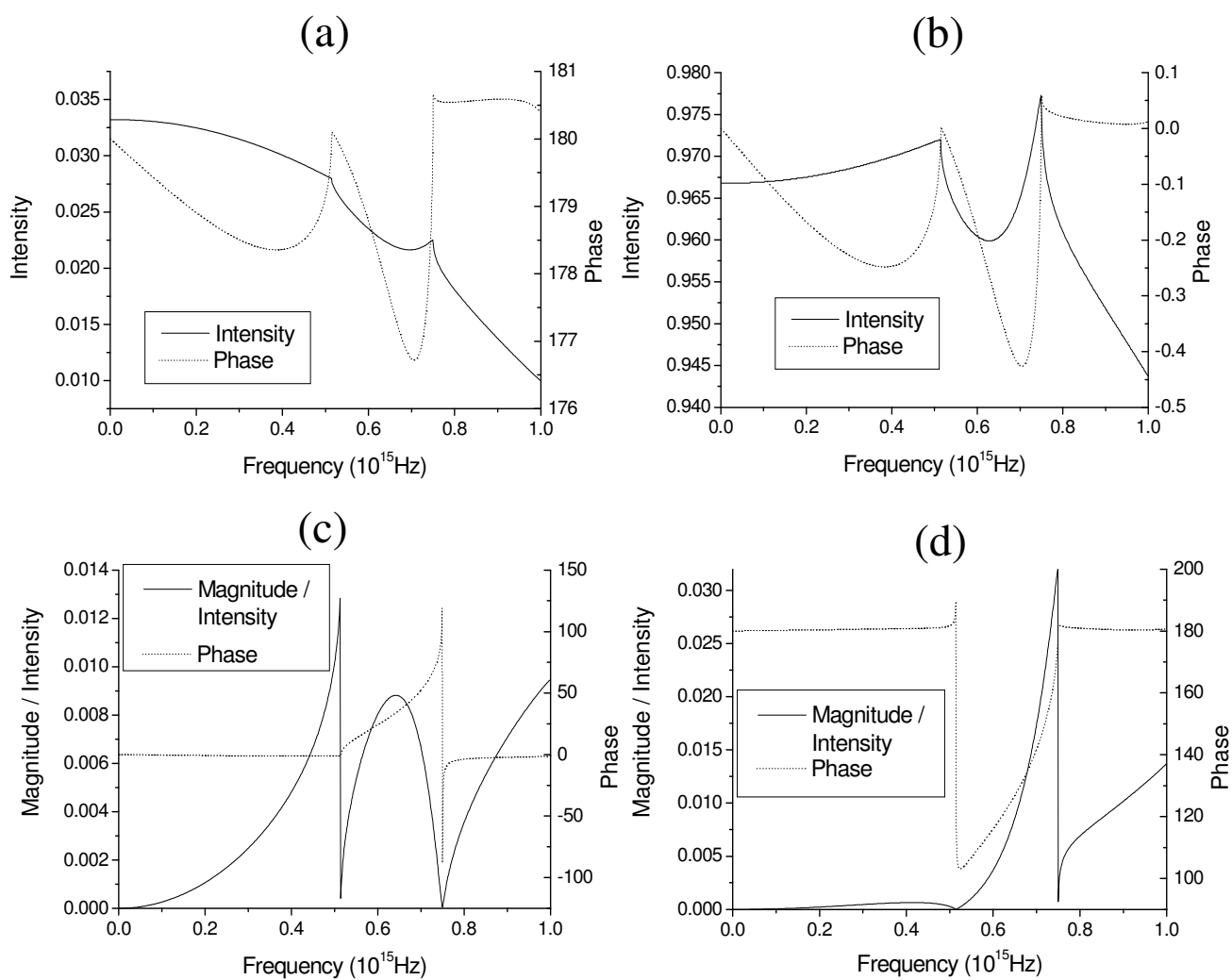
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# Figure 1



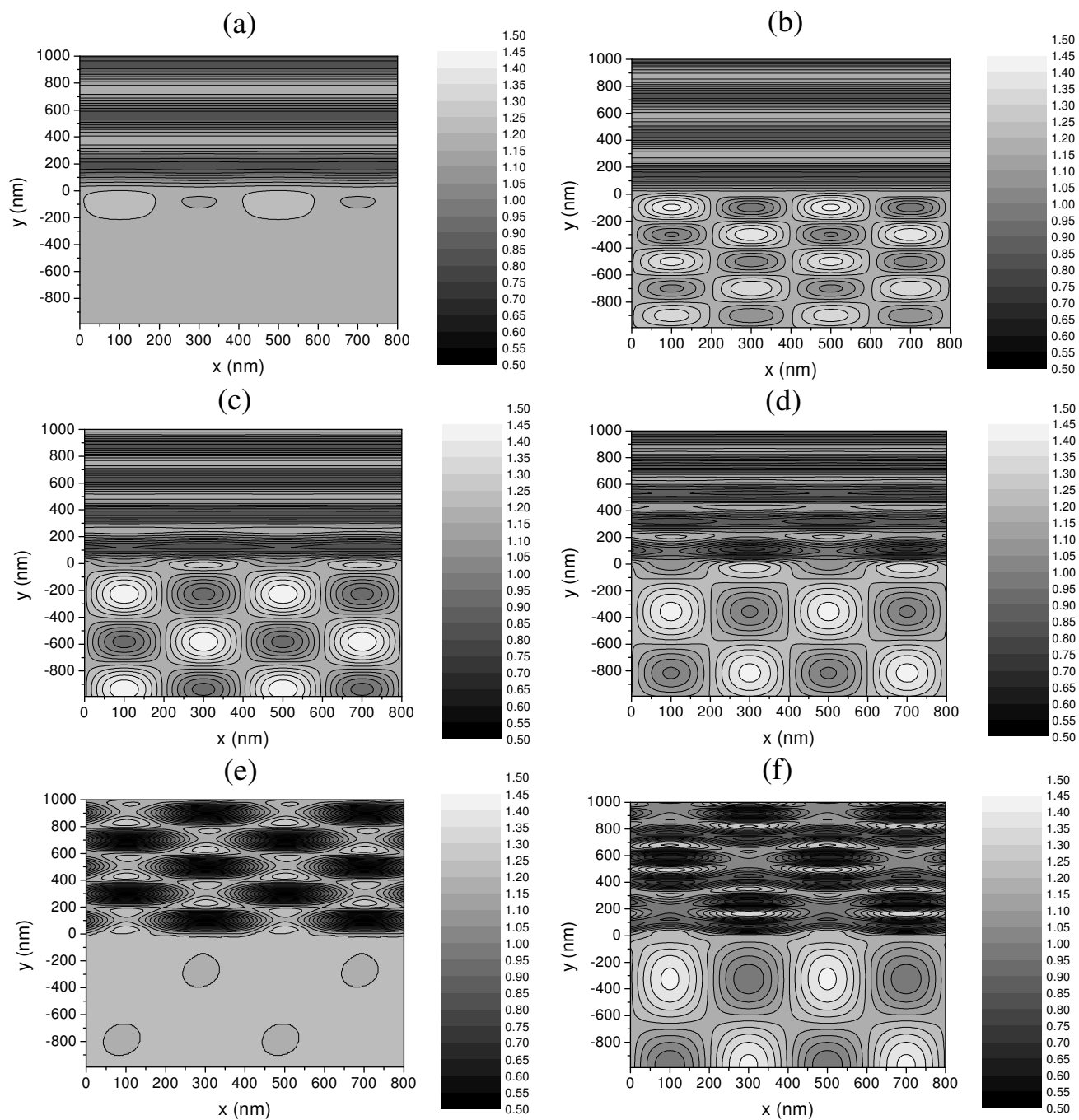
**Figure 1** The optical response of a 400 nm pitch, 25 nm amplitude, sinusoidally corrugated air / SiO<sub>2</sub> grating interface for TM polarised normally incident light. The four orders presented are: a) the zeroth reflected, b) the zeroth transmitted, c) the +1 reflected diffracted, and d) the +1 transmitted diffracted. When the order is real and propagating the intensity of the order is shown, whereas when it is evanescent the magnitude ( $\sqrt{\text{Re}(\mathbf{r}_p)^2 + \text{Im}(\mathbf{r}_p)^2}$ ) is shown.

**Figure 2**



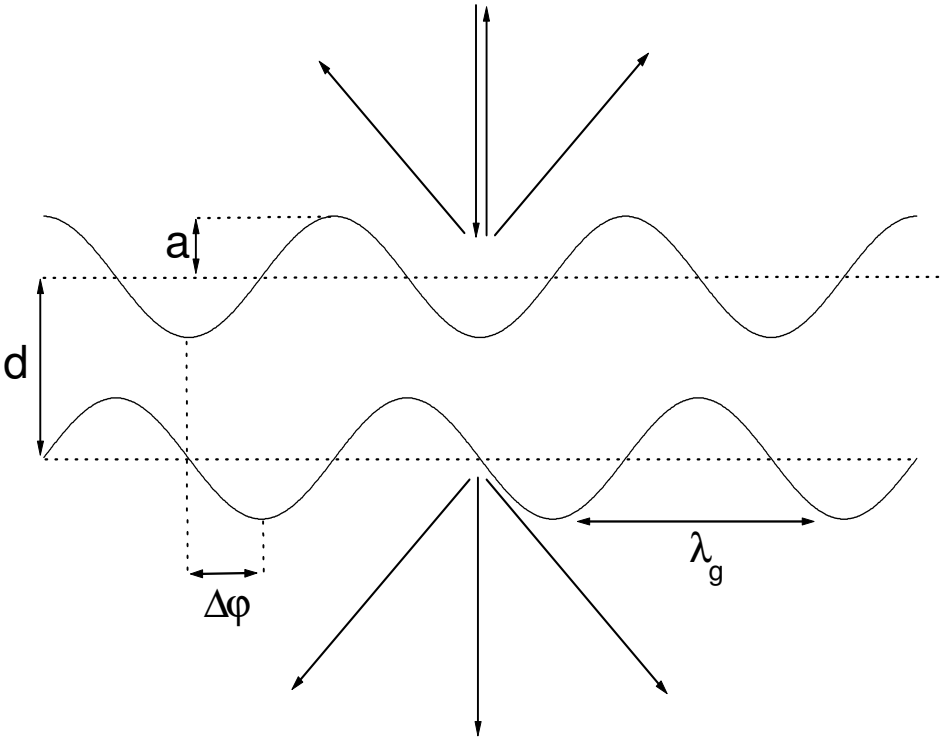
**Figure 2** The optical response of a SiO<sub>2</sub> / Air grating interface, with the same parameters as for the air / SiO<sub>2</sub> interface considered in figure 1. The four orders presented are: a) the zeroth reflected, b) the zeroth transmitted, c) the +1 reflected diffracted, and d) the +1 transmitted diffracted

**Figure 3**



**Figure 3** Time averaged  $H_z$  component of the fields for the air /  $\text{SiO}_2$  interface used for figure 1 for six different frequencies: a)  $f = 0.4 \times 10^{15}$  Hz, b)  $f = 0.514 \times 10^{15}$  Hz, c)  $f = 0.6 \times 10^{15}$  Hz, d)  $f = 0.7 \times 10^{15}$  Hz, e)  $f = 0.75 \times 10^{15}$  Hz, and f)  $f = 0.9 \times 10^{15}$  Hz.

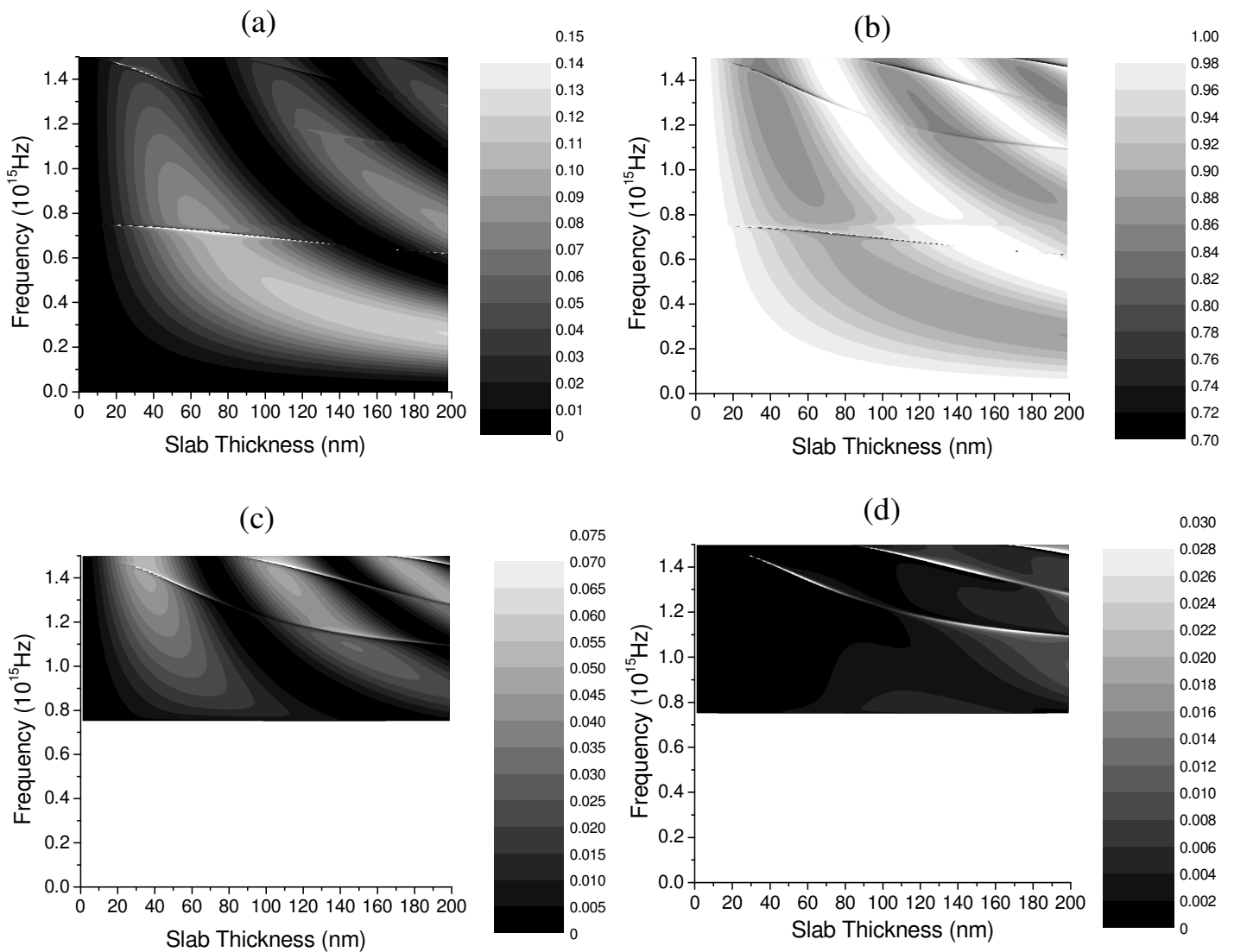
**Figure 4**



**Figure 4.** A schematic of the system under consideration

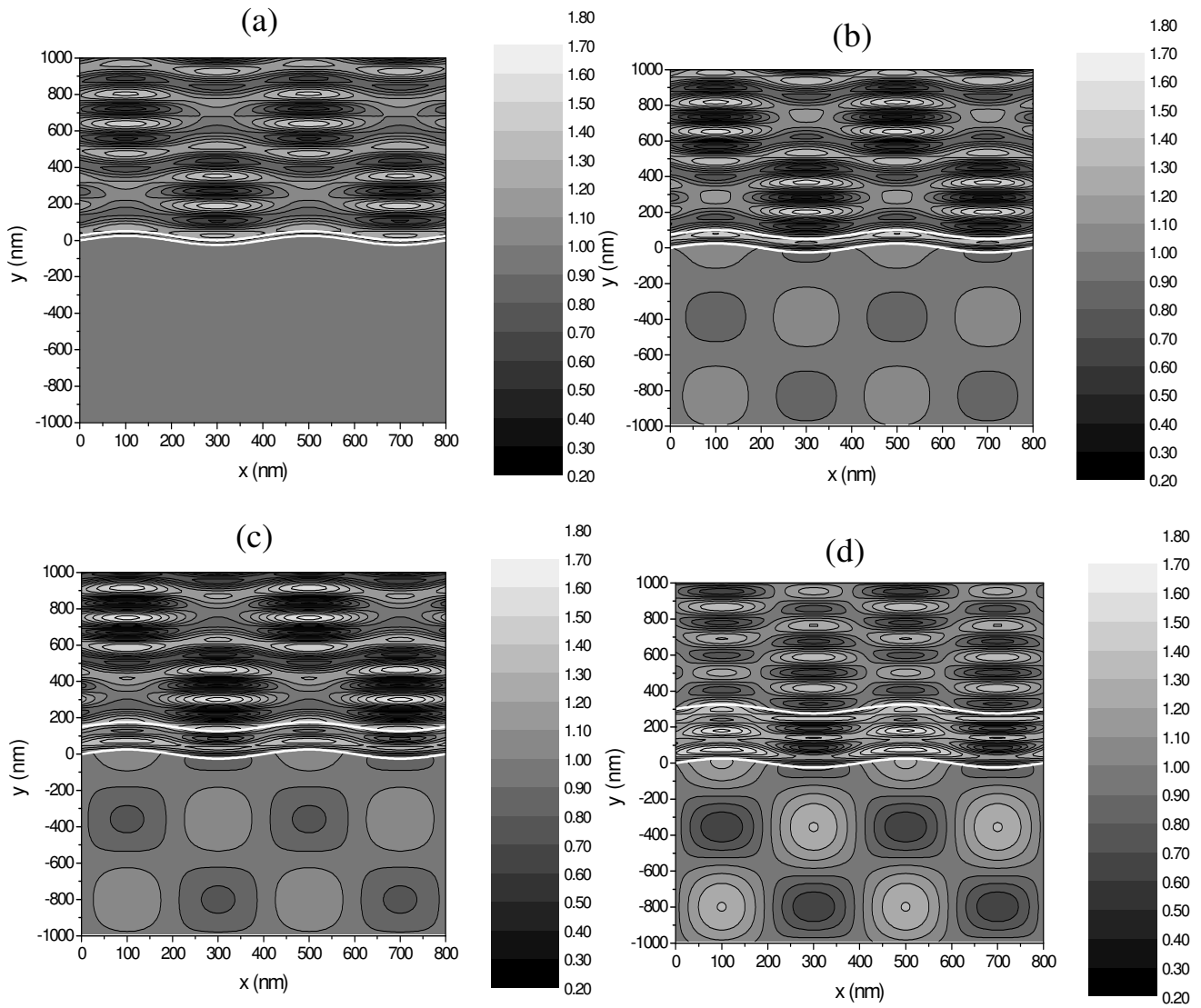


**Figure 5**



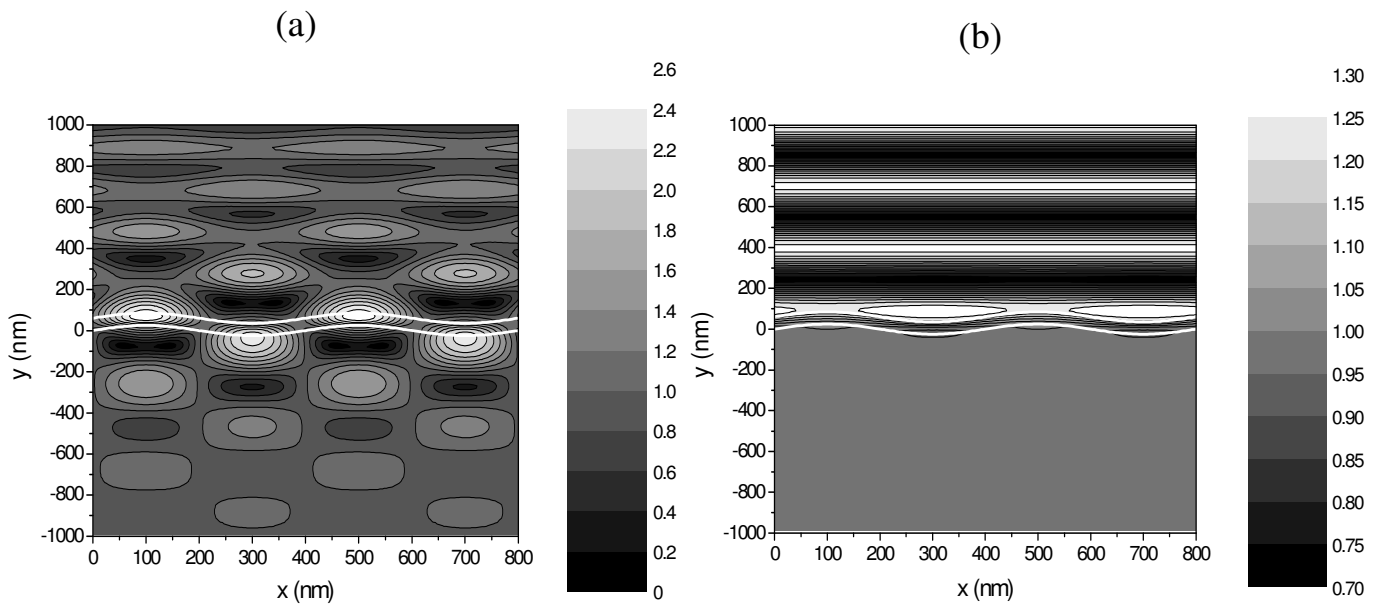
**Figure 5** The optical response of a conformally corrugated thin SiO<sub>2</sub> slab, with air as the bounding media ( $\lambda_g = 400$  nm,  $a = 25$  nm), as a function of the frequency of incident TM polarised light and of the slab thickness. a) the zeroth order reflectivity, b) the zeroth order transmissivity, c) the +1 order reflectivity, and d) the +1 order transmissivity.

**Figure 6**



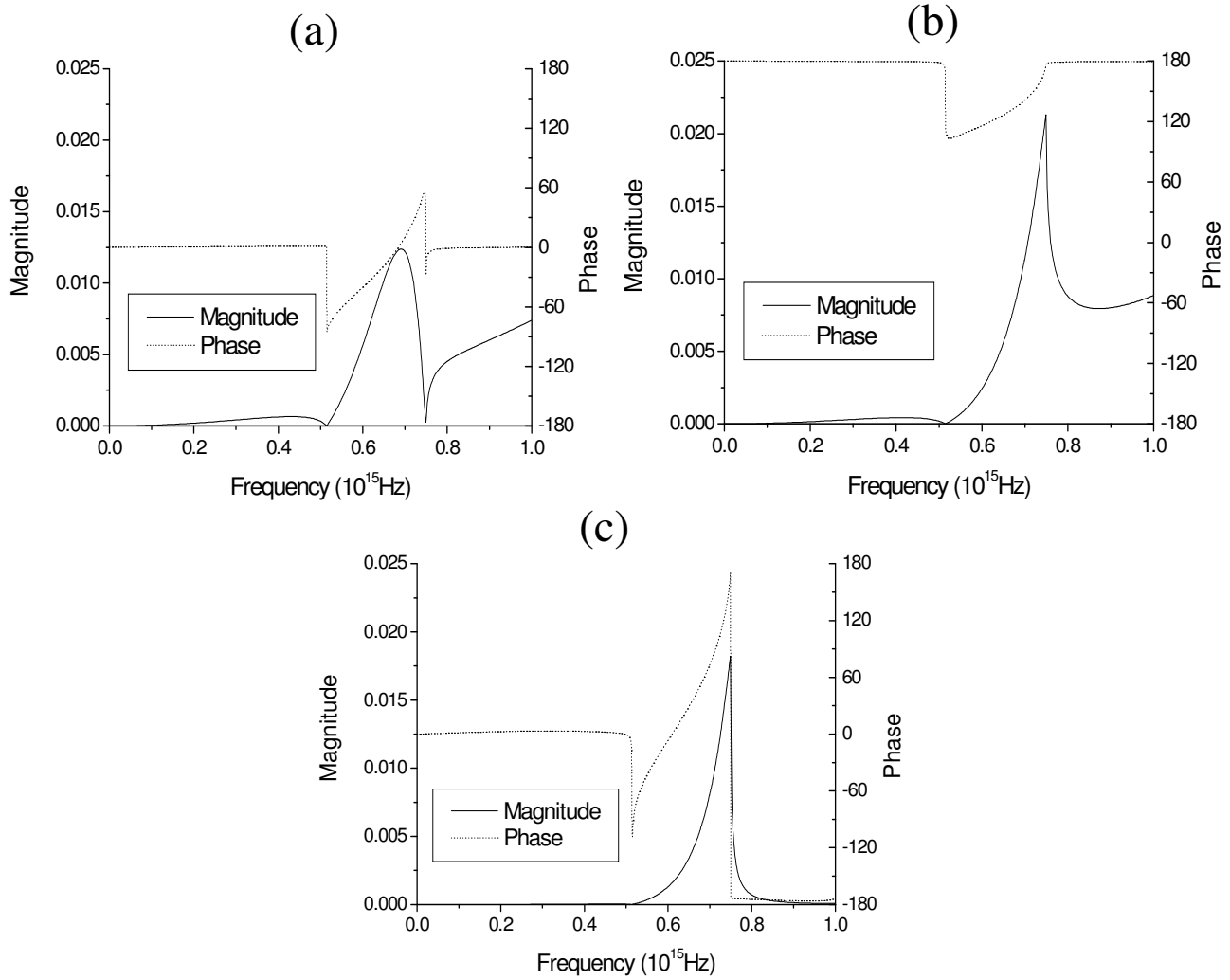
**Figure 6** Time averaged  $H_z$  component of the fields for the system considered in figure 5 for  $f = 1.0 \times 10^{15}$  Hz with different slab thickness. a)  $d = 25$  nm, b)  $d = 75$  nm, c)  $d = 150$  nm, and d)  $d = 300$  nm.

**Figure 7**



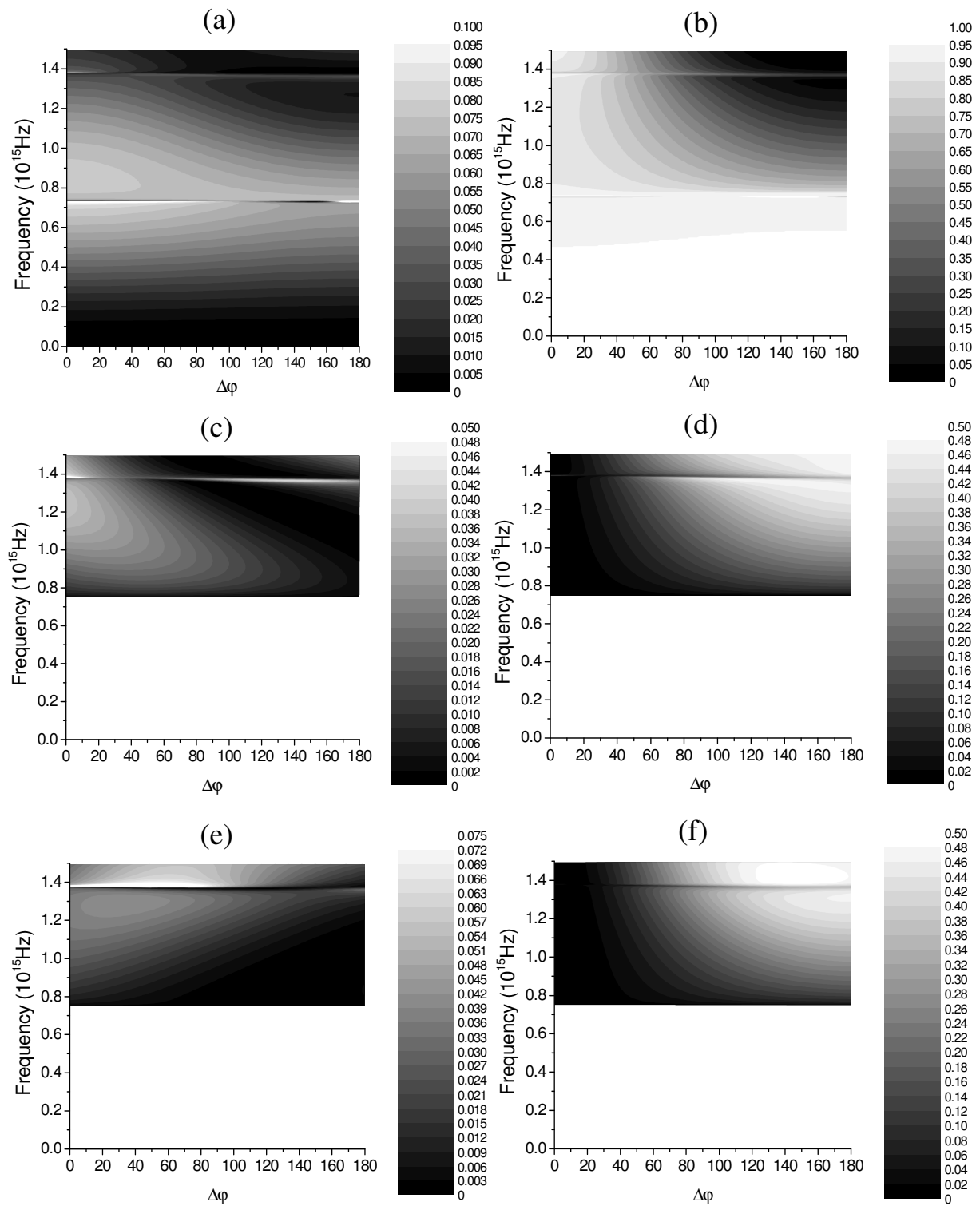
**Figure 7** Time averaged  $H_z$  component of the fields for the system considered in figure 5 for a slab thickness of 60nm. a)  $f = 0.732 \times 10^{15}$  Hz (diffractive in  $\text{SiO}_2$  only), and b)  $f = 0.492 \times 10^{15}$  Hz (non-diffractive in both media).

**Figure 8**



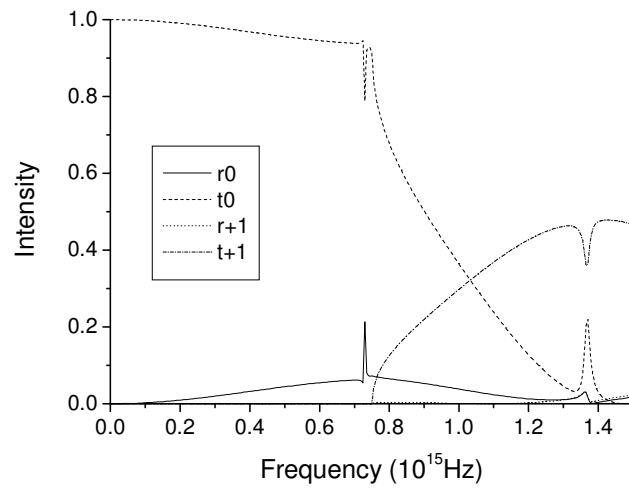
**Figure 8** The magnitude and phase of the amplitude coefficient for the system considered in figure 7 obtained using the ISS method by considering the two diffraction processes separately (as described in the text), for a) diffraction from the top interface, b) diffraction from the bottom interface, and c) the total transmitted fields for the system obtained by combining a) and b).

**Figure 9**



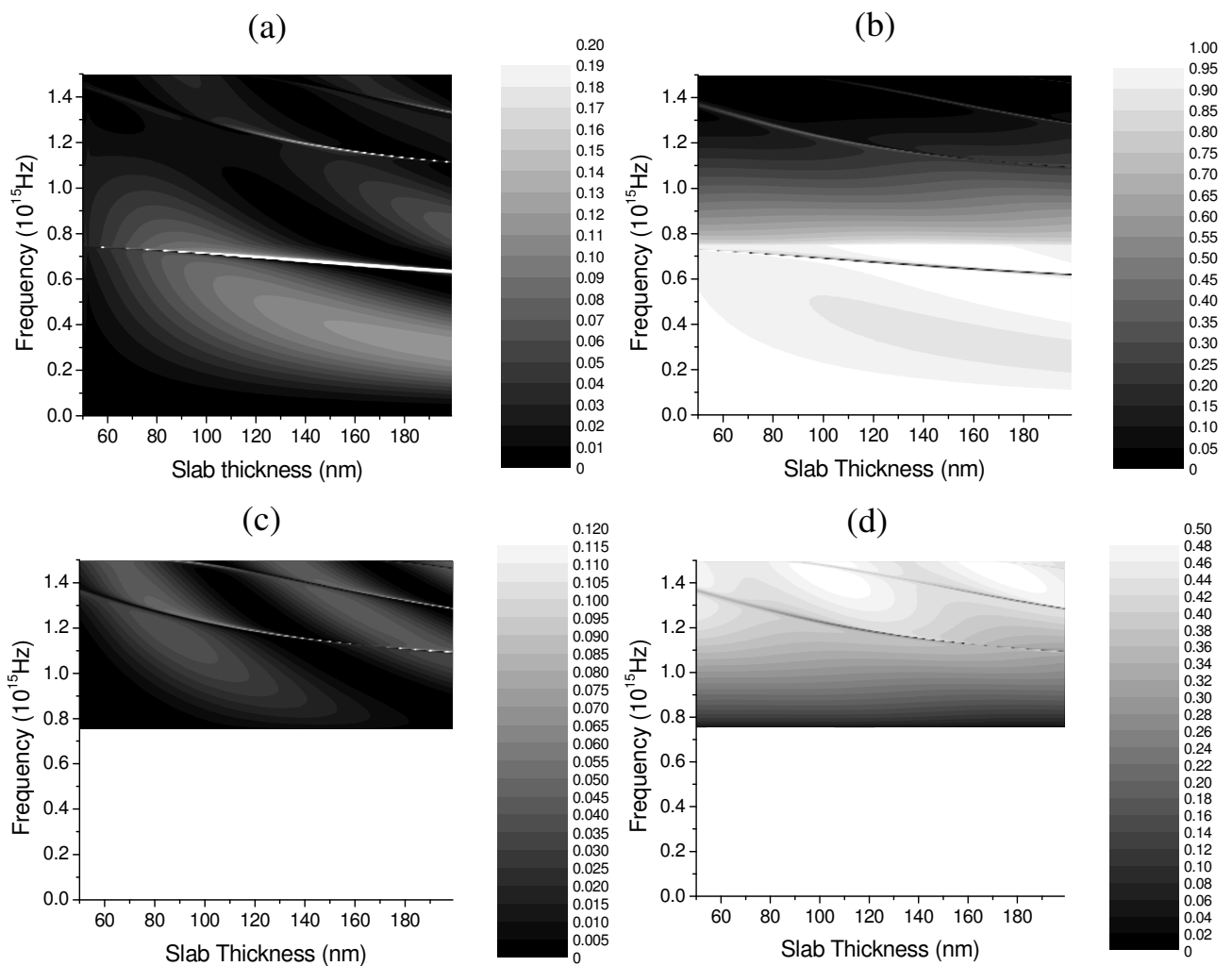
**Figure 9** The optical response of a 60nm thick dielectric slab corrugated on both surfaces with gratings of  $\lambda_g = 400$  nm, and amplitude of 25 nm, as a function of the phase between the corrugations on the two interfaces, and of the frequency of the incident light. a) the reflected zeroth order, b) the transmitted zeroth order, c) the reflected +1 diffracted order, d) the transmitted +1 diffracted order, e) the reflected -1 diffracted order, and f) the transmitted -1 diffracted order.

**Figure 10**



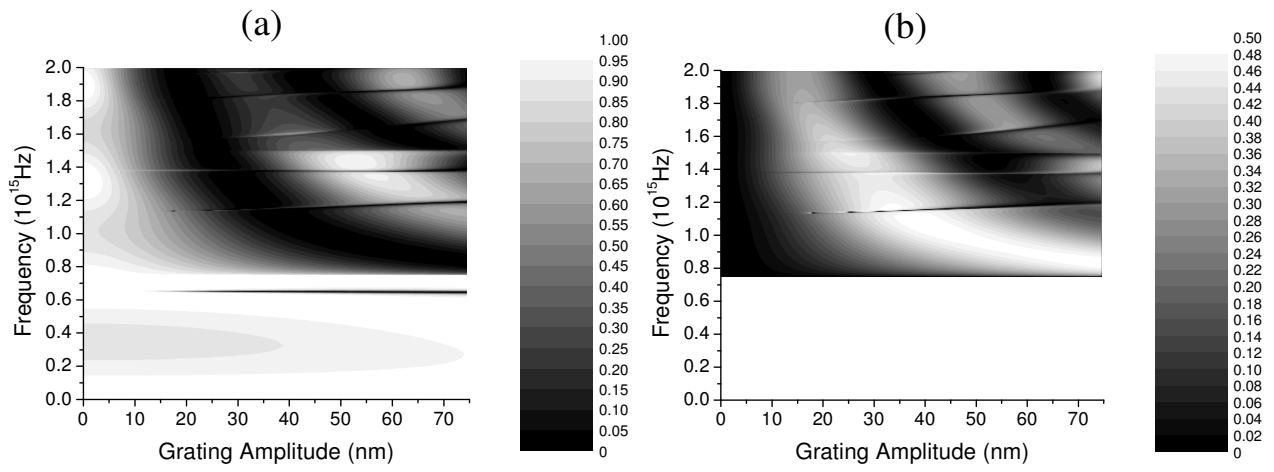
**Figure 10** The intensities of the zeroth reflected and transmitted orders, and of the transmitted and reflected  $\pm$  diffracted orders for a 60 nm thick dielectric slab corrugated on both surfaces with antisymmetric sinusoidal corrugations on each surface of 400 nm pitch and 25 nm amplitude.

**Figure 11**



**Figure 11** The intensities of the various orders from the anti-phase dual interface system considered in figure 10 as a function of frequency and slab thickness. a) the zeroth order reflected, b) the zeroth order transmitted, c) the +1 diffracted reflected, and d) the +1 diffracted transmitted.

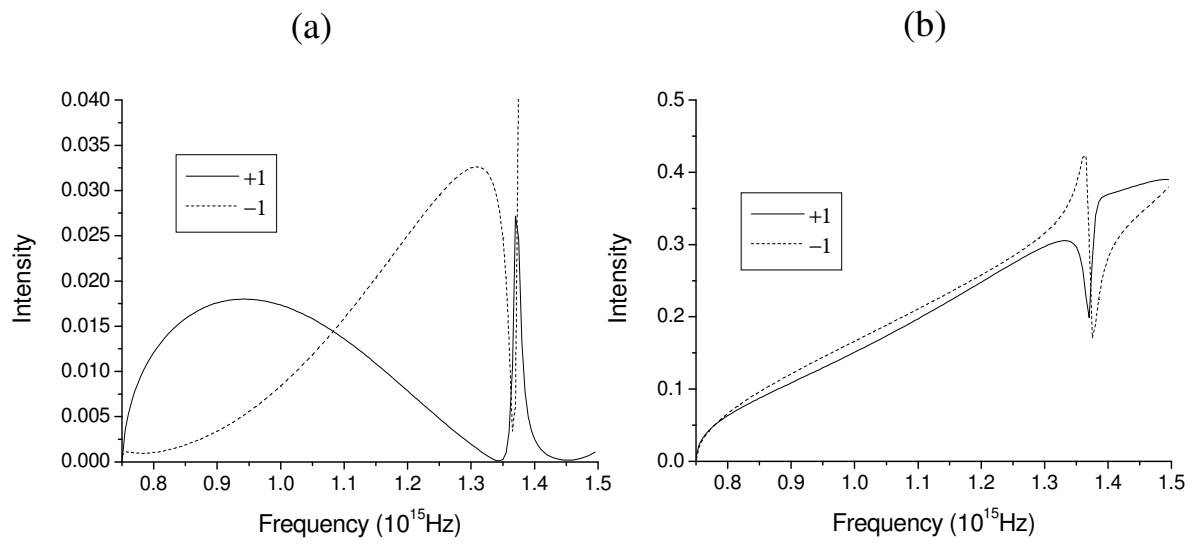
**Figure 12**



**Figure 12** The intensities of the transmitted orders for the system considered in figure 10 as a function of frequency and grating amplitude for a 150 nm thick dielectric slab. a) the zeroth order, and b) the +1 diffracted order.



**Figure 13**



**Figure 13** The intensities as a function of frequency for a two interface corrugated system of thickness 60nm, amplitude 25 nm, and phase difference between the two corrugations of  $90^\circ$ . a) the reflected  $\pm 1$  diffracted orders, and b) the transmitted  $\pm 1$  diffracted orders.