Spin wave interferometer employing a local nonuniformity of the effective magnetic field

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We have investigated scattering of exchange spin waves by a model nonuniformity of the effective magnetic field. In particular, certain profiles of the nonuniformity are characterized by a total transmission of the spin wave intensity while inducing large shifts to the phase of transmitted spin waves. These properties are discussed in the context of potential application within a spin wave logic device—a spin wave interferometer containing such a nonuniformity in one of its branches. We demonstrate limitations imposed upon the size and the speed of operation of such a device by a requirement that it be controlled by an external uniform magnetic field. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2740339]

I. INTRODUCTION

The continuing search for logic paradigms alternative to those based upon semiconductors has led to suggestions that propagating magnetic domains[1–4] or spin waves[5–10] could create a basis for such logic devices. In Mach-Zehnder-interferometer-type logic elements discussed in Refs. 5, 7, and 8, microwave power is split into two waveguide branches containing yttrium iron garnet (YIG) phase shifters. In one of the branches, the phase of the magnetostatic spin wave carrying signals in YIG is modified upon transmission through a region of a nonuniform magnetic field created by electrical current in an underlying stripline. The signals from the two branches of the interferometer are then brought together, and depending upon the induced phase shift, either constructive or destructive interference is observed. Such logic devices have a size of several millimeters, limited by the wavelength of microwaves. For the same frequency, the wavelength of spin waves is orders of magnitude shorter than that of electromagnetic waves, and hence all-spin-wave logic elements, such as those discussed in Refs. 6 and 10, might be much smaller in size than those from Refs. 5, 7, and 9. The nanoscale phase-shifting nonuniformity could be created by introducing a domain wall[6,11] or by a direct local modification of magnetic parameters, e.g., magnetic anisotropy.[12] The challenge is to realize a nonuniformity that would induce 180° phase shift to spin wave while leaving its amplitude unchanged.

The frequency of magnetostatic, exchange, and dipole-exchange spin waves in a 5-nm-thick film of YIG (Ref. 13) is plotted in Fig. 1 as a function of the spin wave wavelength, using the approximate dispersion relations from Ref. 14. One can see that the dispersion of sufficiently short wavelength spin waves (so-called exchange spin waves), which are of primary interest for use within nanoscale all-spin-wave logic devices, is described relatively well even when the dynamic magnetodipole interaction is neglected. The frequency of the exchange spin waves lies in the THz range, which makes them attractive for use in the emerging THz wave technology. Brillouin light scattering[15,16] and scanning Kerr microscopy[17,18] can offer the temporal and spatial resolution

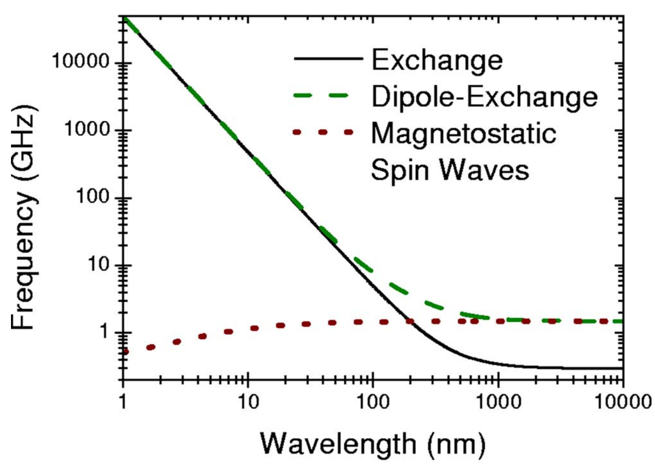

FIG. 1. (Color online) The frequency of exchange, dipole-exchange, and magnetostatic spin waves are plotted as a function of their wavelength for a 5-nm-thick film of YIG, neglecting anisotropy and assuming that the wave vector is parallel to the static magnetization.

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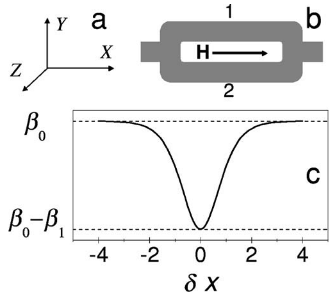

FIG. 2. (a) The coordinate system of the problem is shown. (b) The spin wave interferometer is shown schematically. The values of the magnetic parameters are constant in the reference branch (1), while the values of the parameter of the uniaxial magnetic anisotropy have profiles shown in (c).

that is required for detection of propagating magnetostatic waves. At the same time, it is still to be seen if a similar technique, for example, such as that proposed in Ref. 19, could serve for detection of propagating exchange spin waves.

Here, we report a theoretical investigation of the scattering of exchange spin waves by a nonuniformity of the value of the uniaxial anisotropy and/or of the bias magnetic field. We show that certain profiles of the effective magnetic field are characterized by a total transmission of the spin wave amplitude while inducing sizable shifts to the phase of the transmitted spin wave, as required for spin wave logic devices. Finally, we discuss general limitations inherent to spin wave logic devices of the considered type.

II. THEORY

Let us consider a spin wave interferometer shown in Fig. 2(b), which is similar to that in Ref. 6. The interferometer consists of a spin wave generator, a spin wave guide, and a spin wave detector. The spin wave guide represents a loop with two branches. One branch (1) is made of a stripe of uniform magnetic material and serves as a "reference." The other branch (2) represents a stripe of a magnetic material with a local decrease of the magnetic anisotropy (a "well") and serves as a spin wave "phase shifter." The value of the uniaxial anisotropy in the reference branch is equal to β_0, while that in the phase shifter depends upon the coordinate x along the length of the waveguide as

$$\beta(x)=\beta_0-\frac{\beta_1}{\cosh^2 x/d}, \tag{1}$$

where β_1 and d are the "depth" and the characteristic "width" of the well, respectively. The graph of function (1) is shown in Fig. 2(c). The signal registered by the detector results from interference of the two spin waves transmitted through the phase shifter and the reference. The spontaneous magnetization and the exchange parameter are assumed to be equal to the same values M_0 and α in both branches, respectively. The discussion below applies to the straight parts of the interferometer only, while the phase shifts induced to spin waves within its curved parts are equal in both branches. Within the straight regions, the easy axis of the uniaxial anisotropy and the directions of the internal magnetic field $\mathbf{n}H$ and of the static magnetization $\mathbf{n}M_0$ are all parallel to the x axis.

The dynamics of magnetization $\mathbf{M}(\mathbf{r},t)$ is described by the Landau-Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t}=-g[\mathbf{M}\times\mathbf{H}_E], \tag{2}$$

with the effective magnetic field defined in the exchange approximation as

$$\mathbf{H}_E=[H+M_0\beta(x)]\mathbf{n}+\alpha\Delta\mathbf{M}, \tag{3}$$

where g is the gyromagnetic ratio ($g>0$).

Let us consider small deviations $\mathbf{m}(\mathbf{r},t)$ of the magnetization from the ground state, i.e., a uniform magnetization parallel to the easy axis. For this purpose, we represent magnetization as

$$\mathbf{M}(\mathbf{r},t)=M_0\mathbf{n}+\mathbf{m}(\mathbf{r},t), \quad \text{where } |\mathbf{m}|\ll M_0. \tag{4}$$

Introducing variable $m_+=m_x+im_y$ and seeking solutions in the form of harmonic waves $m_+(r,t)=m(x)\exp\{i\omega t\}$, we obtain the following linearized equation for $m(x)$:

$$\frac{d^2m(x)}{dx^2}+\frac{1}{\alpha}\left(\Omega-h-\beta_0+\frac{\beta_1}{\cosh^2(x/d)}\right)m(x)=0, \tag{5}$$

where $\Omega=\omega/gM_0$ and $h=H/M_0$ are the dimensionless frequency and magnetic field, respectively. This equation can be reduced to the hypergeometrical Gauss equation,[20] whose general solution may be written as[21]

$$\begin{aligned} m=&C_1F[0.5(\lambda+i\kappa d);\,0.5(\lambda-i\kappa d);\,0.5;\\ &-\sinh^2(x/d)]+iC_2\sinh(x/d)F[0.5(1+\lambda\\ &+i\kappa d);\,0.5(1+\lambda-i\kappa d);\,1.5;\,-\sinh^2(x/d)], \end{aligned} \tag{6}$$

where C_1 and C_2 are arbitrary constants, F is the hypergeometrical function,[20] and λ is determined from equation $\beta_1 d^2=\alpha\lambda(\lambda-1)$. The asymptotic value of the wave number κ is given by

$$\kappa=\sqrt{\frac{\Omega-h-\beta_0}{\alpha}}.$$

Far enough from the well, solution (6) can be asymptotically represented as

$$m\sim\begin{cases}\exp\{i\kappa x\}+R\exp\{-i\kappa x\} & x\to-\infty\\ T\exp\{i\kappa x\} & x\to+\infty\end{cases}, \tag{7}$$

where factors R and T have meaning of the *amplitude* coefficients of reflection and transmission of spin waves, respectively, and can be written as[21]

$$R=\frac{\exp\{2i\Theta\}+\exp\{2i\Xi\}}{2},\quad T=\frac{\exp\{2i\Theta\}-\exp\{2i\Xi\}}{2}, \tag{8}$$

where Θ and Ξ are given by

$$\Theta=\arg\frac{\Gamma(i\kappa d)\exp\{-i\kappa d\ln 2\}}{\Gamma(0.5\lambda+i0.5\kappa d)\Gamma[0.5(1-\lambda)+i0.5\kappa d]},$$

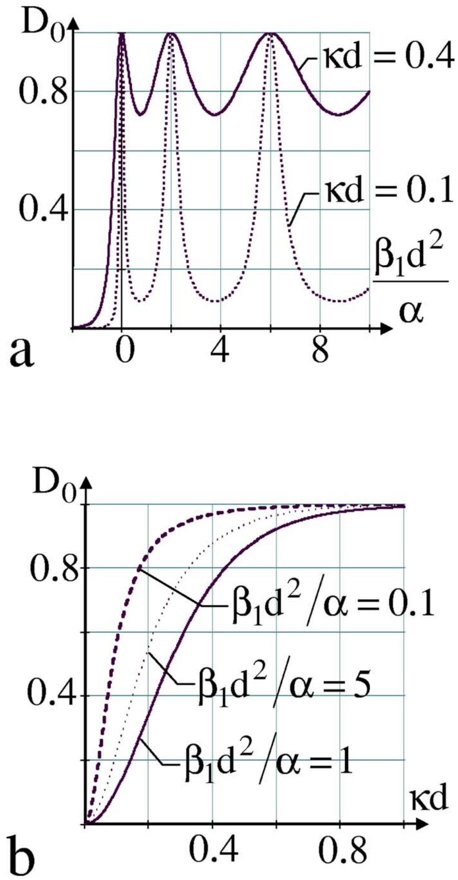

FIG. 3. (Color online) The dependence of the intensity transmission coefficient D_0 upon the wave number of the signal carrying spin wave and upon the parameters of the well is shown.

$$\Xi = \arg\frac{\Gamma(i\kappa d)\exp\{-i0.5\kappa d\ln 2\}}{\Gamma[0.5(\lambda-1)+i0.5\kappa d]\Gamma(1-0.5\lambda+i0.5\kappa d)}. \tag{9}$$

Using these formulas, the *intensity* coefficients of reflection and transmission of spin waves can also be calculated. In particular, for the intensity transmission coefficient D_0, one can obtain

$$D_0 = \sinh^2(\pi\kappa d)[\sinh^2(\pi\kappa d)+\sin^2(\pi\lambda)]^{-1}. \tag{10}$$

The dependence of D_0 upon the wave number and the parameters of the well are illustrated in Fig. 3. Note that $D_0 \equiv 1$, when condition

$$\lambda = 1,2,3,\ldots \tag{11}$$

is fulfilled, in which case spin waves pass the well without reflection. Well profiles satisfying condition (11) correspond to so-called reflectionless Pöschl-Teller well potential.[21,22] Phases (9) are then related as

$$\Theta - \Xi = \frac{\pi}{2}. \tag{12}$$

Even in the absence of reflection, the phase of the transmitted spin wave is modified by the well, and so the spin wave intensity at the output of the interferometer $D_0^{(i)}$, which results from the interference of the waves transmitted through the phase shifter and the reference, is equal to

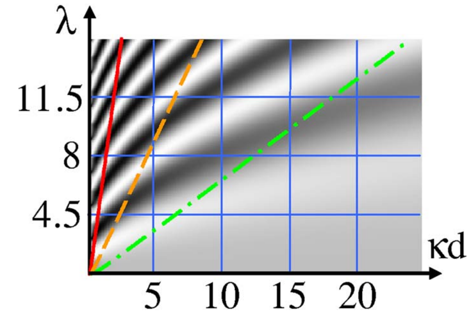

FIG. 4. (Color online) The output of the spin wave interferometer $D_0^{(i)}(\kappa d,\lambda)$ is plotted as a grayscale map. The white and black correspond to 0 and 1, respectively. The dependence of $D_0^{(i)}$ upon κ and λ for the other parameters fixed is represented by the horizontal and vertical cross sections of the surface $D_0^{(i)}(\kappa d,\lambda)$, respectively. The dependences of $D_0^{(i)}$ upon α and d are represented by the projections of the cross sections of the surface $D_0^{(i)}(\kappa d,\lambda)$ along curves defined by Eq. (15) upon the vertical and horizontal axes. Three such curves are plotted for $\beta_0=5$ and $\beta_1=1$, while $\Omega-h$ is equal to 5.2 (red, solid), 5.7 (orange, dash), and 7 (green, dash-dot).

$$D_0^{(i)} = \frac{1}{4}|1+T|^2. \tag{13}$$

For the reflectionless well profiles, we can write using Eqs. (8), (12), and (13),

$$D_0^{(i)} = \sin^2(\Xi). \tag{14}$$

III. DISCUSSION

Let us now discuss in some detail the dependence of the output of the interferometer upon the parameters of the phase shifter, the frequency of the spin wave, and the applied magnetic field. From Eqs. (8), (9), and (13), it is easy to see that the output is a function of self-similar variables κd and λ. To take advantage of this, let us plot $D_0^{(i)}(\kappa d,\lambda)$ as a two-dimensional grayscale map shown in Fig. 4. In Fig. 4, the black and white correspond to a destructive and constructive interference of the spin waves from the two branches, respectively. However, a full transmission or a full extinction of the signal can be observed only when the amplitudes of the two spin waves are equal, that is only for integer values of λ (horizontal lines), unless the spin wave in the reference branch is accordingly attenuated. The dependence of $D_0^{(i)}$ upon κ (and hence upon Ω, h, and β_0) and λ (and hence upon β_1) for the other self-similar parameters fixed is represented by the horizontal and vertical cross sections of the two-dimensional surface $D_0^{(i)}(\kappa d,\lambda)$, respectively. To study the dependence of $D_0^{(i)}$ upon α and d, we have to take into account that both κd and λ depend upon another self-similar variable, α/d^2. Excluding the latter from the expressions for κd and λ, we obtain equation of lines, different points of each of which are obtained by varying α/d^2 for fixed values of Ω, h, β_0, and β_1

$$\kappa^2 d^2\beta_1 = (\Omega-h-\beta_0)\lambda(\lambda-1). \tag{15}$$

The dependences of $D_0^{(i)}$ upon α and d are then represented by the projections of the cross sections of the two-dimensional surface $D_0^{(i)}(\kappa d,\lambda)$ by line (15) upon the vertical and horizontal axes. In doing so, one needs to note that $\alpha = \beta_1 d^2/\lambda(\lambda-1)$.

In order to use the interferometer as a magnetic-field-controlled logic element, one must be able to shift from the black to the white along one of the horizontal lines corresponding to an integer λ by varying the value of h and hence κd. The required variation of the bias magnetic field depends upon the derivative

$$\partial\kappa/\partial h = -(\partial\Omega/\partial h)/(\partial\Omega/\partial\kappa) = -(\partial\Omega/\partial h)/v_g, \tag{16}$$

where v_g is the group velocity of the spin wave. For exchange spin waves, $\Omega = h + \beta_0 + \alpha\kappa^2$, and so, $\partial\kappa/\partial h = -1/2\alpha\kappa$. Thus, the shorter the wavelength of the spin waves is, the larger is the change of the bias magnetic field required to change the output of the interferometer from "0" to "1," which is also apparent from Fig. 4.[23] This may present a serious limitation for the desired miniaturization of spin wave logic devices of this type. Furthermore, since the group velocity of spin waves appears in the denominator of Eq. (16), one may also need to compromise between the efficiency of the magnetic field control and the speed of operation of such a device. Due to the general nature of Eq. (16), one may also notice that the most efficient control by the bias magnetic field is achieved for dipole-exchange spin waves in the backward volume geometry,[14] which have a characteristic minimum in their frequency at a particular finite value of the wave number. For example, magnetostatic backward volume spin waves were used in the millimeter-sized spin wave logic devices from Ref. 7. Alternatively, in a nanoscale device, one could use materials with periodically modulated magnetic properties (e.g., the anisotropy or the exchange parameter) instead of the homogeneous materials considered here. Such materials (so-called *magnonic crystals*) have recently been subject to increased attention in the context of the emerging field of *magnonics*.[24–29] In magnonic crystals, the dispersion of spin waves contains so-called band gaps and regions of reduced group velocity in their vicinity.[26,28] Hence, the use of magnonic crystals within spin wave interferometers and of spin waves with frequency near the band-gap edges as signal carriers could lead to more efficient and miniature spin wave logic devices. However, investigation of such devices is beyond the scope of the present paper.

IV. SUMMARY

In summary, we have investigated the scattering of exchange spin waves by a localized nonuniformity of the effective magnetic field. We have shown that certain profiles of the effective magnetic field are characterized by a total transmission of the amplitude while inducing large phase shifts required for operation of spin wave logic devices. At the same time, we have demonstrated a serious crosstalk between the size and the speed of operation of such devices and the efficiency of their control by an applied magnetic field.

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